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Private Money Creation with Safe Assets and Term Premia*

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Abstract

It has been documented that an increase in the demand for safe assets induces the private sector to create more money-like claims. Focusing on private repos backed by U.S. Treasury securities, I show that an increase in the demand for safe assets leads to a *decreases* in the issuance of Treasury repos. The intuition is that Treasury securities already function as a safe asset, thus in terms of safe asset creation, private Treasury repos are neutral. In the model, Treasury repos are beneficial because they shift risk (i.e. term premia) from relatively risk averse households to a more risk tolerant financial sector, which issues repos to finance its portfolio. When the demand for safe assets increases, Treasury securities are reallocated to households, reducing the amount of Treasury repo issued by the financial sector. By contrast, Treasury repos created by the Federal Reserve's RRP program—a safe asset created by the public sector—increase with the demand for safe assets. I show the data supports the model's main predictions.

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1 Introduction

The growth of the shadow banking system has been cited as one of the important precursors of the 2007–09 financial crisis. In particular, some researchers have argued that the shortage of publicly produced safe assets induced the private sector to create more of them in the form of short-term debt. The newly created short-term debt supposedly satisfied the demand for safe assets because it also provides a *convenience yield*—broadly defined as a non-pecuniary benefit from holding safe/liquid assets.[1] Although the financial sector's role in creating private safe assets was important to satisfy demand, it came at the cost of increased financial fragility. Specifically, the financial sector's increased reliance on short-term "safe" debt left it exposed to runs, which ultimately manifested itself in the global financial crisis.

An important component of the aforementioned mechanism depends on the financial sector's ability to fill the public sector's shortfall in safe asset supply. In effect, Krishnamurthy and Vissing-Jorgensen (2015) show that safe and liquid government debt crowds out the financial sector's short-term debt. Along the same lines, Sunderam (2015) provides empirical evidence that prior to the crisis private issuance of asset backed commercial paper responded positively to an increase in the demand for safe assets. Both of these findings indirectly imply that the growth of the shadow banking system was driven, in part, by the need to satisfy the demand for safe assets.

In this paper I find that the sensitivity of private short-term debt issuance depends on the debt's characteristics. Specifically, I provide evidence that the issuance of private repurchase agreements (repos) backed by U.S. Treasuries *decreases* as the demand for safe assets *increases*. This challenges the view of the existing literature but can easily be rationalized by noting that U.S. Treasuries already satisfy the demand for safe assets. That is, in terms of safe assets supply, U.S. Treasuries and repos backed by U.S. Treasuries serve the same purpose. I argue that an increase in the demand for safe assets reallocates more U.S. Treasuries to the non-financial sector, reducing the amount of repo issuance. Moreover, I find that the Federal Reserve's overnight and term reverse repo program

[1] See Gorton (2016) for a detailed description of the literature's history and terminology.

(henceforth labeled RRP) exhibits the opposite sensitivity. That is, the Federal Reserve increases the supply of its repos as the demand for safe assets increases. The intuition is that if the Federal Reserve chooses not to sell its U.S. Treasury holdings, the RRP program is the only way it can distribute safe assets directly to the non-banking sector.

The empirical results stem from analyzing U.S. Treasury repo outstanding in the tri-party repo market between 2009 and 2016. I find that changes in aggregate and individual firm repo outstanding decreases as a measure of the safe asset convenience yield increases. I also find that the size of the Federal Reserve's RRP program increases as a measure of the safe asset convenience yield increases. The convenience yield is measured by the spread between the four week Treasury bill (T-bill) and the four week overnight index swap (OIS). Given that T-bills are publicly produced short term safe assets and the OIS is a contractual agreement which promises a risk free payoff, the spread measures the premium for holding safe assets, that is, the safe asset convenience yield. The results for both private repo issuance and the RRP is statistically significant in several specifications.

The paper's mechanism gives rise to a natural question: If U.S. Treasuries already satisfy the demand for safe assets, why are U.S. Treasury repos issued in the first place? I argue that repos allow risky assets to be held by a more risk tolerant financial sector, while supplying safe assets to the rest of the economy. More specifically, this paper assumes that both short and long term government bonds satisfy the demand for safe assets, but long term bonds are risky in the short run, that is, they have term premia.[2] Repos allow term preima to be transferred to the financial sector while at the same time fill households' demand for safe assets. Thus, in equilibrium both public and private forms of safe assets coexist.

To highlight this intuition, this paper provides a simple theoretical model in the spirit of Krishnamurthy and Vissing-Jorgensen (2015). The model considers an economy with two sectors, households and banks, which purchase two types of government securities: short-term bonds (T-

[2]It may seem contradictory to assume safe assets can have a risky payoff. As in Gorton (2016), this paper defines a safe asset as an asset that pay par with a very high probability. U.S. Treasury bonds have this property. But in the short run, the value of these assets can change because of term premia. A more detailed discussion of the modeling assumptions is given in subsection 2.4.

bills) and long-term bonds (Treasury bonds). Both securities can satisfy the demand for safe asset, but T-bills are risk free while Treasury bonds are risky in the short run. Households are risk averse and enjoy an additional benefit for holding both types of safe assets. That is, households consume a non pecuniary return for owning either T-bills or Treasury bonds. Banks are risk neutral and do not enjoy the additional benefit of holding safe assets, but have the technology to produce safe assets backed by government securities, that is, repos. The resulting equilibrium involves both households and banks holding Treasury bonds, and banks issuing repos backed by their portfolio. When households' demand for safe assets increases, they increase their Treasury holdings, reducing the amount of repos issued by banks. I later modify the model to introduce a central bank which can issue repos directly to households and show conditions under which the behavior of the central bank's repo issuance is the opposite to that of private banks. The intuition is that even though the central bank's repo issuance can be backed by Treasuries, the model does not allow the central bank to sell them. Therefore, the only way the central bank can satisfy the demand for safe assets is by allowing households to increase their takeup at the central bank's repo facility.

The main theoretical contribution of the paper is to consider long term safe assets with term premia. Previous literature recognizes that short term and long term safe assets are different. Krishnamurthy and Vissing-Jorgensen (2012) and Krishnamurthy and Vissing-Jorgensen (2015) highlight this difference by assuming that each asset provides a different type of non pecuniary safe asset benefit. In contrast, the model in this paper places short term and long term safe assets on the same "safe asset footing" by assuming they are perfect substitutes, but recognizing that long-term safe assets have term premia. Thus risk averse agents have to balance the benefits from holding safe assets with their inherent riskiness. Even though both types of assets are equally useful to satisfy the demand for safe assets, in equilibrium long-term assets are more effective because they trade at a discount. That is, term premia makes long-term safe assets more useful to satisfy the demand for safe assets.

The model also gives predictions on the sensitivity of private repo issuance to changes in the supply of public safe assets. An increase in T-bills reduces households' marginal demand for safe assets, resulting in a decrease in private repo issuance. However, an increase in Treasuries has an ambiguous effect. On the one hand, it reduces households' marginal demand for safe assets, decreasing the amount of private repo issuance (as with T-bills). On the other, it increases banks Treasury holdings, increasing the amount of private repo issuance. The data confirms that changes in short term T-bills outstanding decrease private repo issuance, and that changes in Treasury outstanding increase private repo issuance.

The paper is structured as follows. The end of section 1 gives a brief overview of the literature. Section 2 characterizes the model, showing the intuition under what conditions repo issuance is negatively related to an in households' demand for safe assets, and the extension which considers central bank repo issuance. Section 3 provides empirical evidence from the U.S. tri-party market and also highlights how the sensitivity of the Federal Reserve's RRP program differs from the rest of the private market. Finally, Section 4 gives some concluding remarks and outlines future work to be explored.

Brief Literature Review

The aforementioned story hinges on a number of components. The first is that the financial sector can create privately produced safe assets. This is in the spirit of traditional banking models such as Diamond and Dybvig (1983), Holmstrom and Tirole (1998), and Gorton and Pennacchi (1990). Although these papers differ in the rationale behind the creation of private short term debt—be it for liquidity risk sharing or to alleviate asymmetric information frictions—the main theme is the private creation of liquid, safe assets.

Another necessary ingredient is the existence of a convenience yield for safe assets, a characteristic which T-bills and U.S. Treasuries have. Typically modeled in reduced form, a growing literature argues that safe, money-like assets provide benefits above and beyond their risk/return trade off. For example, Greenwood et al. (2015) study the monetary premium associated with short term government debt and Krishnamurthy and Vissing-Jorgensen (2012) show that the U.S. Treasury has benefited from issuing long term bonds at reduced yields because of their safe asset status.

In addition, this paper's mechanism relies on the ability of privately produced safe assets to satisfy the economy's demand for safe assets. In effect, Gorton et al. (2012) document that in the U.S over the past 60 years the share of safe assets relative to GDP has been constant, but over the past 30 years the share of safe asset provided by the shadow banking sector has grown. Relatedly, Carlson et al. (2014) study whether the central bank can reduce systemic risks by providing short term safe assets, effectively crowding out excessive private safe asset creation.

Conceptually, this paper is similar to the analysis Krishnamurthy and Vissing-Jorgensen (2012) and Sunderam (2015) which directly quantify the sensitivity of privately produced safe assets to the demand for safe assets. I find that the characteristics of privately produced safe assets matter in understanding how the financial sector can fill the safe asset gap.

Finally, this paper builds and expands on several insights documented in Nagel (2016). Nagel shows that the the safe asset premium depends on the opportunity cost of holding money.[3] That is, the convenience yield is largely determined by the level of rates. This paper takes convenience yield dynamics as exogenous, and studies changes in the relative value and holdings of safe assets. But the model outcome of this paper also suggests that the level of rates is important in determining the safe asset premium. Nagel also provides evidence that the elasticity of substitution between money like claims is equal to 1, that is, money and money like claims behave like perfect substitutes. The setup of this paper assumes that different money-like assets are perfect substitutes when satisfying the demand for safe assets, placing them all on the same "safe asset footing".[4] Finally, Nagel studies central banks' implementation of *interest on reserves* and shows that the relationship between the convenience yield and the level of rates remains unchanged. This suggests that market segmentation created by the banking sector's unique ability to hold reserves has a strong effect. I show that the Federal Reserve's RRP program expands to satisfy the demand for safe assets, alluding to the programs' ability to circumvent the banking sector and provide safe assets directly to the economy.

[3]What this current paper calls safe assets, Nagel calls near-money assets.

[4]This assumption is discussed in detail in subsection 2.4.

2 Model

The theoretical model in the paper is in the spirit of Krishnamurthy and Vissing-Jorgensen (2015) with an important difference: one of the safe assets is risky, generating a non-trivial portfolio problem for risk averse agents.

2.1 Setup

The model consists of two periods, $t \in \{0, 1\}$. In the initial period agents choose their optimal portfolio, how many contracts to trade between each other, and how much to consume. In the final period asset payoffs are realized, contracts are settled, and agents consume their final wealth. The model consists of two types of agents: households and financial firms which are called "banks". Both households and banks start with an initial endowment to purchase an exogenous supply of securities and any endogenously created contracts. Banks have the technology to use their assets to issue collateralized loans (i.e., repos) to households, which are the model's endogenous contracts.

2.1.1 Assets & Contracts

There are two exogenously supplied securities in the economy. One security is called T-bill and pays off a fixed return R_T in the second period with certainty. The second security is called U.S. Treasury (henceforth simply called Treasury) and has a random return $\tilde{R}$ in the second period, which follows a distribution $F(\cdot)$ with mean μ. The return on $\tilde{R}$ is the only risk in the economy. In $t = 0$ there are a fixed amount of Θ_T T-bills and Θ_U Treasury securities in the economy.

Banks can issue loans called repos which are collateralized by either T-bills or Treasuries. For simplicity, I assume there is no limited liability and banks can always repay their debts, making repos risk free. That is, repos pay off a fixed return R_{RP} in the second period with certainty.

In addition to providing a store of value and a risky payoff, all assets can provide an additional convenience yield for holding them. That is, a portfolio of T-bills, Treasuries, and repos (θ_T, θ_U, RP)

can provide a convenience yield which is a function of the total *moneyness* of the portfolio,

$$M = R_T\theta_T + \mu\alpha_U\theta_U + R_{RP}\alpha_{RP}RP \tag{1}$$

In this characterization, an asset's contribution to the convenience yield is represented by its expected payoff in period 1. Returns and expected returns are determined in equilibrium. In the general setup, T-bills' contribution to the portfolio's moneyness is normalized to one, and Treasuries and repos contributions are scaled by α_U and α_{RP}, respectively. Only households enjoy the convenience yield of holding these assets which is detailed below. Equation 1 implicitly assumes all assets are perfect substitutes to fill the demand for safe assets.

2.1.2 Households

Households have a fixed initial endowment of e which they use to purchase assets and consume. They are risk averse and have time separable utility, discounting consumption in the future period by β. Give a pair of consumption plans $(c_0, \tilde{c}_1)$, households utility function takes the following form,

$$U^H = u(c_0) + \beta\mathbb{E}(u(\tilde{c}_1 + v(M;\eta))) \tag{2}$$

where $v(M;\eta)$ captures households added utility for holding a portfolio with moneyness M. As in Krishnamurthy and Vissing-Jorgensen (2015), and others, safe assets' convenience yield is modeled in reduced from. I assume that both u and v satisfy $u', v' > 0$ and $u'', v'' < 0$. The parameter η serves to increase and decrease the households preference for liquid assets exogenously, which serves to do comparatives statics. I assume $v'_\eta, v'_\eta > 0$, that is, the level and marginal convenience yield increases with η. Given households' portfolio of $(\theta_T^H, \theta_U^H, RP^H)$, households consumption in both periods is,

$$c_0 = e_0 - \theta_T^H - \theta_U^H - RP^H$$
$$\tilde{c}_1 = R_T\theta_T^H + \tilde{R}\theta_U^H + R_{RP}RP.$$

Households will choose $(\theta_T^H, \theta_U^H, RP)$ to maximize equation (2) resulting in the following Euler equations,

$$\mathbb{E}(\tilde{S})(1 + v'(M;\eta))R_T = 1 \tag{3}$$
$$\mathbb{E}(\tilde{S}(\tilde{R} + \alpha_U v'(M;\eta)\mu)) = 1 \tag{4}$$
$$\mathbb{E}(\tilde{S})(1 + \alpha_{RP} v'(M;\eta))R_{RP} = 1 \tag{5}$$

where $\tilde{S} = \beta\frac{u'(\tilde{c}_1 + v(M))}{u'(c_0)}$. Note that if $\alpha_{RP} < 1$ then $R_T < R_{RP}$, and if $\alpha_{RP} = 1$ then $R_T = R_{RP}$. To simplify the characterization of the equilibrium, I assume that each asset has the same contribution to the portfolio's moneyness, that is, in equation (1) $\alpha_U = \alpha_{RP} = 1$, thus $R_T = R_{RP}$.[5]

2.1.3 Banks

The model considers one representative risk neutral bank with the technology to issue repos at a cost $C(\cdot)$, which is paid in $t = 1$, with $C(0) = 0, C', C'' > 0$. Given the banking sector's risk preference, they provide a service to the economy by holding risky Treasuries and issuing repos. That is, households are able to shift risk to banks who are more willing to take it, yet still reap the convenience yield from the contracts they issue. The representative bank starts with an initial

[5] Details for this assumption are given in subsection 2.4

endowment w and wants to maximize dividends in $t = 0, 1$ which take the following form,

$$\begin{aligned} Dvd_0 &= w + RP - \theta_T^B - \theta_U^B \\ Dvd_1 &= -R_{RP}RP + R_T\theta_T^B + \tilde{R}\theta_U^B - C(RP) \end{aligned}$$

where θ_T^B are the bank's holdings in T-bills, θ_U^B are banks holding in Treasuries, and RP are the amount of repos issued. For simplicity I assume that banks aren't subject to limited liability, but they must satisfy their initial budget constraint,

$$w + RP \geq \theta_T^B + \theta_U^B. \tag{6}$$

In the general version of the model, repo haircuts can be incorporated. That is, the amount of repos issued must be lower than a fraction of the banks portfolio value,

$$RP \leq (1 - hc_T)\theta_T^B + (1 - hc_U)\theta_U^B. \tag{7}$$

For the baseline model, I will consider that banks have a large enough endowment so that this restriction does not bind. This assumption is reasonable because in the model banks are risk neutral, and given households' pricing of securities, banks will always want to purchase more risky assets. With a high enough initial endowment, the marginal decision to issue repos depends on the costs and benefits of issuing them, and not on a bank's ability to pay for haircuts.

Formally, the banks solve the following problem,

$$\max_{\{\theta_T^B, \theta_U^B, RP\}} Dvd_0 + \beta\mathbb{E}(Dvd_1)$$

subject to,

$$\begin{aligned} w + RP &\geq \theta_T^B + \theta_U^B \\ (1 - hc_T)\theta_T^B + (1 - hc_U)\theta_U^B &\geq RP \\ \theta_T^B, \theta_U^B, RP &\geq 0. \end{aligned}$$

Denoting λ and ξ the Lagrange multiplier associated with the budget and haircut constraint, respectively; the bank's problem gives rise to the following first order conditions,

$$-1 + \beta R_T - \lambda + (1 - hc_T)\xi \leq 0 \tag{8}$$

$$-1 + \beta\mu - \lambda + (1 - hc_U)\xi \leq 0 \tag{9}$$

$$1 - \beta R_{RP} - \beta C'(RP) + \lambda - \xi \leq 0 \tag{10}$$

By focusing on equilibria where banks either hold Treasuries or T-bills, that is $\max\{\beta R_T, \beta\mu\} > 1$, from inequality (8) or (9) it is clear that a slack budget constraint ($\lambda = 0$) and a binding haircut constraint ($\xi > 0$) cannot be an equilibrium: The bank would always prefer to increase it's portfolio holdings in these assets, making the haircut constraint slack.

The interesting case is when the budget constraint binds and the haircut restriction is slack ($\lambda > 0, \xi = 0$). In this case, if $R_T \geq \mu$ then the bank holds as many T-bills that its budget constraint allows and does not issue repos. In effect, in this case $\lambda = \beta R_T - 1$ and, since $R_{RP} = R_T$, from (10) $RP^* = 0$.[6] Alternatively, if $\mu > R_T$ then the bank holds as many Treasuries as it can and issues repos, i.e, inequality (10) binds. In effect, in this case $\lambda = \beta\mu - 1$, using (9) gives,

$$C'(RP) = \mu - R_{RP} \tag{11}$$

that is, the marginal cost to issue repos is equal to the expected return of holding Treasuries minus

[6] Recall that since $C(0) = 0$ and $C' > 0$.

the cost of financing them. Note that under these conditions, the bank never holds T-bills, i.e., inequality (8) is slack.

Therefore, in case $\mu \leq R_T$, the banks optimal strategy is to not issue repos and hold the following portfolio,

$$\theta_T^{B*} = w, \quad \theta_U^{B*} = 0.$$

Alternatively, if $\mu > R_T$, the banks optimal strategy is to issue repos according to equation (11), and hold the following portfolio,

$$\theta_T^{B*} = 0, \quad \theta_U^{B*} = w + RP.$$

Note that the above optimal strategies are still feasible when $w = 0$ and with no repo haircuts $hc_U = 0$.

2.2 Equilibrium

Since T-bills and repos are both risk free, and contribute equally to the portfolio's moneyness, households treat both assets identically. In addition, equation (4) can be rewritten as,

$$\mu - R_T = -cov(\tilde{S}, \tilde{R})R_T \tag{12}$$

The optimal strategies characterized in subsection 2.1 lead to the following equilibrium,

Proposition 1. *If $\Theta_U - w > 0$ and $\mathbb{V}(\tilde{R})$ is sufficiently high enough, equilibrium returns are given by equations (3) and (12) with $R_T = R_{RP}$. The amount of repos issued by the bank is given by $C'(RP^*) = \mu - R_T$, and households hold Θ_T T-bills and $\Theta_U - (w + RP^*)$ Treasuries.*

Proof. The strategies characterized in subsection Proposition 1 are optimal if $\mu - R_T > 0$ and $\beta\mu > 1$. Because of equation (12), the first condition holds if $cov(\tilde{S}, \tilde{R})$ is negative. Given u's concavity, this is true if $\tilde{c}_1$ is correlated with $\tilde{R}$, that is, households holdings of Treasury securities is positive. Reasoning by contradiction, assume that $\mu - R_T \leq 0$. In that case, the bank's optimal strategies imply zero repo issuance and zero Treasury holdings. Thus households hold all Treasury securities because of market clearing, leading to a contradiction.

In the case when $\mu - R_T > 0$, then the household's holdings of Treasury securities is $\Theta_U - (w + RP^*)$ which is positive since $\Theta_U - w > 0$ and $C'(RP^*) = \mu - R_T$ with $C' > 0$ and $C(0) = 0$.

Finally, from equation (12) it is easy to see that for a high enough $\mathbb{V}(\tilde{R})$, $\beta\mu > 1$.

□

Proposition 1 characterizes the equilibrium where the bank and households split the total amount Treasuries, the bank issues repos to households, and households hold all T-bills. This equilibrium captures the model's main economic intuition: In spite of the exogenous cost of issuing repos, there are welfare gains in doing so because banks are better at bearing risk than households.

In addition, denoting $R_f = \frac{1}{\mathbb{E}(\tilde{S})}$ as the canonical one period risk free rate, equation (3) can be rewritten as,

$$R_T - R_f = -v'(M; \eta) R_T. \tag{13}$$

that is, the difference between the return of the tradable risk free rate, relative to the theoretical risk free rate, is negatively related to the marginal benefit of holding safe assets scaled by the level of short term rates, as in Nagel (2016). Equation 13 motivates the measurement of the convenience yield in Section 3.

The equilibrium characterized in Proposition 1 can shed light into how prices and repo issuance respond to changes in the convenience yield, modeled through changes in η. In addition, the model can shed light onto how the supply of public safe assets can affect the equilibrium outcome. To simplify the model, I assume agents have CARA utility and the issuance cost function takes a simple quadratic form: $C(RP) = \frac{1}{2}c \times (RP)^2$ with $c > 0$. Under this specific setup, the model gives the following comparative statics,

Proposition 2. *If households have CARA utility with risk aversion γ, $C(RP) = \frac{1}{2}c \times (RP)^2$, and* *the parameter conditions of Proposition 1 hold with $\gamma(\Theta_T + \Theta_U - w) := \gamma\kappa > 1$, then repo issuance has the following comparative statics,*

$$
\begin{aligned}
\frac{\partial RP^*}{\partial \eta} &= \frac{-1}{|D|}\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)\mathbb{E}(\tilde{S})R_T\left\{\gamma\kappa v_\eta v'' + v'_\eta\left(-\gamma\kappa(1+v') + \frac{1}{R_T}\right)\right\} < 0 \\
\frac{\partial RP^*}{\partial \Theta_T} &= \frac{-1}{|D|}\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)\mathbb{E}(\tilde{S})R_T v''(1+\gamma\kappa) < 0 \\
\frac{\partial RP^*}{\partial \Theta_U} &= \frac{-1}{|D|}\frac{1}{c}\left\{-\gamma\left(-\gamma\kappa(1+v') + \frac{1}{R_T}\right)\left[\hat{V} - \left(\frac{\mu}{R_T} - 1\right)^2(1+v')R_T\right]\right. \\
&\quad \left. + \mathbb{E}(\tilde{S})R_T v''\left(-\gamma\hat{V}\kappa + \left(\frac{\mu}{R_T} - 1\right)\left(\frac{\mu}{R_T} + \gamma\kappa\right)\right)\right\}
\end{aligned}
$$

where $\hat{V} := \mathbb{E}(\tilde{S}(\tilde{R} - \mu)^2)$, and $|D| < 0$ is the determinant of the Jacobian matrix of partial derivatives of (3) *and* (12) *with respect μ, R_T.*

Proof. See Appendix □

Proposition 2 provides the model's main results: Increases in convenience yield, measured through η, reduces the amount of repo issuance. The intuition is that there is a reallocation of U.S. Treasuries from the bank to households, resulting in a decrease in repo issuance. In addition, the proposition shows how public safe asset issuance affects repo issuance. Increases in the amount of T-bills decreases repo issuance while increases in the amount of Treasuries has an ambiguous sign. This last result highlights the two forces at play: Increases in Treasury issuance satisfies the demand for safe asset, but it also increases gains from risk shifting.

2.3 Introduction of Central Bank Repos

The model of the previous subsection considered 3 assets in the economy: 2 publicly produced assets and 1 privately produced asset. In this subsection, the model introduces a fourth assets: repos issued by a central bank which households can buy directly. The new asset is intended to model the Federal Reserve's policy tool introduced in 2013 called the RRP.[7]

In this extension households and the bank must also choose what fraction of their portfolio to invest in the central bank's repo facility, θ_{CB}, which earns the policy rate R_{CB} and contributes to the portfolio's moneyness with the following specification,

$$M = R_T\theta_T + \mu\alpha_U\theta_U + R_{RP}\alpha_{RP}RP + R_{CB}\alpha_{CB}\theta_{CB}.$$

Solving for household optimal portfolio choice gives the same pricing equations as the previous subsection (equations (3) - (5)), plus an additional one:

$$\mathbb{E}(\tilde{S})(1+\alpha_{CB}v'(M;\eta))R_{CB} = 1.$$

As before, if $\alpha_{CB} > 1$ then $R_{CB} < R_T$, and if $\alpha_{CB} = 1$ then $R_{CB} = R_T$. Turning to the bank's portfolio problem, if $\mu > R_{CB}$ it is unattractive for the bank to invest in repos issued by the central bank. Therefore, as before, the bank and households split the total amount Treasuries, the bank issues repos to households, and households hold the remaining assets in the economy.

To simplify the analysis, I consider a case in which only long term bonds exist (i.e., $\Theta_T = 0$) and $\alpha_U = \alpha_{RP} = \alpha_{CB} = 1$. In this case, the private market repo rate is equal to the central bank's repo rate, and the characterization of the equilibrium collapses into two equations,

$$\mathbb{E}(\tilde{S})(1+v'(M;\eta))R_{CB} = 1 \tag{14}$$

$$\mu - R_{CB} = -cov(\tilde{S},\tilde{R})R_{CB}. \tag{15}$$

Although the equilibrium characterization looks identical as in the previous subsection, there is an important different: The policy rate is exogenous and amount invested in the central bank's repo

[7] This policy tool allows the Federal Reserve to interact directly with cash investors, giving them an option to deposit funds with the central bank at a predetermined policy rate, effectively circumventing the traditional banking sector.

program is an equilibrium outcome. That is, in this model, the central bank sets the policy rate R_{CB} and the total amount of "takeup" at the central bank's facility is determined in equilibrium, denoted by Θ_{CB}. The other equilibrium variable is the risky asset's expected return μ.[8] Focusing on symmetric equilibrium, so that the individual households' portfolio choice coincides with aggregate takeup at the central bank's repo facility, the following proposition characterizes the equilibrium:

Proposition 3. *If $\Theta_U - w > 0$ and $\mathbb{V}(\tilde{R})$ is sufficiently high enough, the equilibrium expected return for the risky asset and the total amount of takeup in the central bank's repo facility is given by equations* (14) *and* (15)*, with $R_{CB} = R_{RP}$. The amount of repos issued by the private bank is given by $C'(RP^*) = \mu - R_{CB}$, and households hold all of the central bank's repo issuance and $\Theta_U - (w + RP^*)$ Treasuries.*

Proof. The proof is identical to the proof of Proposition 1 by relabeling Θ_T with Θ_{CB} and R_T with R_{CB}, and using equations (14) and (15). □

Proposition 3 characterizes the equilibrium where the central bank sets the interest rate on its repo facility and households choose how much to invest in it. Both households and banks share the supply of Treasuries and banks issue a positive amount of repos. This equilibrium shows that both the central bank and the private bank can issue repos to satisfy households' demand for safe asset. From this equilibrium we can characterize how takeup in the facility and private repo issuance responds to changes in the demand for safe assets. As in the previous subsection, I consider the simplifying case when households have CARA utility and $C(RP) = \frac{1}{2}c \times (RP)^2$ with $c > 0$.

Proposition 4. *If households have CARA utility with risk aversion γ, $C(RP) = \frac{1}{2}c \times (RP)^2$, and the parameter conditions of Proposition 3 hold with $v'' > 0$ sufficiently small, then central bank repo*

[8]To simplify notation of this version of the model, I will use the same variable to denote the risky asset's expected return. I will also use the same variable for the amount of private repo issuance.

issuance and private repo issuance has the following comparative statics,

$$\begin{aligned}
\frac{\partial RP^*}{\partial \eta} &= \frac{-1}{|\overline{D}|}\frac{1}{c}\left(\frac{\mu}{R_{CB}}-1\right)\mathbb{E}(\tilde{S})R_{CB}^2\left\{\gamma v_\eta v'' - v_\eta'\left((1+v')+\frac{1}{R_{CB}}\right)\right\} < 0 \\
\frac{\partial \Theta_{CB}}{\partial \eta} &= \frac{-1}{|\overline{D}|}\left\{v_\eta\left(-\frac{\gamma^2}{c}\left[\hat{V}-\left(\frac{\mu}{R_{CB}}-1\right)^2(1+v')R_T\right]-\right.\right. \\
&\quad v''\gamma R_{CB}\mathbb{E}(\tilde{S})\left(\frac{\mu}{R_{CB}}-1\right)\frac{\partial M}{\partial \mu}-\left.\frac{\gamma}{R_{CB}}\right)+ \\
&\quad \left.\mathbb{E}(\tilde{S})R_{CB}v_\eta'\left(\frac{\gamma\hat{V}}{c}+\gamma\left(\frac{\mu}{R_{CB}}-1\right)(1+v')\frac{\partial M}{\partial \mu}+\frac{1}{R_{CB}}\right)\right\}
\end{aligned}$$

where $\hat{V} := \mathbb{E}(\tilde{S}(\tilde{R}-\mu)^2)$, *and* $|\overline{D}| < 0$ *is the determinant of the Jacobian matrix of partial derivatives of* (14) *and* (15) *with respect* μ, Θ_{CB}.

Proof. See Appendix □

Proposition 4 shows how both private and public repo issuance responds to changes in households' demand for safe assets. The effect over private bank repo issuance is similar to the one in Proposition 2: An increase in the demand for safe assets results in a decrease in private repo issuance. Again, the intuition is that there is a reallocation of U.S. Treasuries from the private bank to households, resulting in a decrease in private repo issuance. The sensitivity of takeup at the central bank's repo facility has two components: One is related to a level increase in the demand for safe assets, and the other is related a marginal increase in the demand for safe assets. Equation (13) shows that movements in the spread between a safe asset and a risk free contract capture changes in the marginal demand for safe assets. Thus, to relate Proposition 4 to the empirical analysis, the focus should be on the sensitivity of Θ_{CB} to movements in v', that is, v'_η.[9] In Proposition 4 the term accompanying v'_η in $\partial\Theta_{CB} \,/\, \partial\eta$ is positive, implying that an increase in v' will increase takeup in the central bank's facility.[10] The following subsection gives more details on the model's main assumptions and predictions.

[9]See Section 3 for details on the empirical measurement of the demand for safe assets.

[10]See appendix for details

2.4 Discussion of Modeling Assumptions and Equilibrium Outcome

An important difference between this paper and the existing literature is how long term safe assets contribute to the economys total supply of safe assets. Specifically, equation (1) assumes that safe assets are prefect substitutes when satisfying the demand for safe assets. Moreover, the model focuses on the case where the contribution of each asset is assumed to be equal: $\alpha_U = \alpha_{RP} = 1$. In contrast, much of the existing literature makes important differences on how safe assets satisfy the demand for safe assets. These differences can be summarized by observing how they enter into the utility function:

$$u(\tilde{c} + v(M)),$$

where differences arise in the characterization of M and v. Greenwood et al. (2015) assume that M only consists of short term safe assets. In their framework, short maturity is a necessary condition to satisfy the demand for safe assets. This seems like a reasonable condition when thinking about cash investors that may have a high preference for short maturity portfolios, such as money market mutual funds. But other types of cash investors may not have such strong maturity preferences. For example, securities lenders are active participants in cash markets and also hold long term illiquid portfolios.[11]

Importantly, Krishnamurthy and Vissing-Jorgensen (2012) provide strong evidence that long term U.S. Treasuries have a significant premium due to their safe asset status. In their theoretical setup, they assume that investors' demand for safe assets can be expressed by a short term component $v_{ST}(\cdot)$) and a long term component $v_{LT}(\cdot)$).[12] Therefore, the total moneyness of a portfolio is split into short term safe assets M_{ST} and long term safe assets M_{LT}. This view is consistent with the notion that the demand for safe assets stem from different types of liquidity needs. For example, the need to store value, a characteristic of short term safe assets, may differ significantly from the need to post collateral to raise funding quickly, a characteristic of long term

[11]Other examples may include corporate cash managers, sovereign wealth funds, and insurance companies.

[12]In their paper, $v_{LT}(\cdot)$ is written as $\mu(\cdot)$. I adopt a different notation to avoid confusion with the long term asset's expected return.

safe assets. But the way in which safe assets satisfy demand is still an open empirical question. In this paper, I assume that short and long term safe assets are equally useful in satisfying household's non-pecuniary benefit from holding them. Rather than consider different ways in which safe assets provide additional benefits, this paper recognizes the inherent difference between short and long term safe assets, namely, that long term safe assets are risky in the short run. That is, I place all safe assets on the same "safe asset footing", but recognize they have other differences which affect investors' choice to hold them.

In addition to using one functional form to capture the benefits from holding different types safe assets, I assume that they are prefect substitutes in playing this role. Sunderam (2015) assumes each assets' contribution to a portfolios moneyness is aggregated with a constant elasticity of substitution strictly greater than one. Imperfect safe asset substitutability may capture different motives behind holding safe assets, but it also introduces a modeling convenience which may be unwanted: there must be a positive supply of all safe assets. In the context of this paper, it would imply that repos are necessary from a first principals perspective. In contrast, in this model repos arise endogenously because they provide an important service to the economy, namely, to transfer risk from risk averse agents to more risk tolerant ones. In addition, Nagel (2016) finds that money and near-money assets are perfect substitutes. This paper takes that concept one step further by assuming that that result holds between other asset classes which provide safe asset benefits.

Turning to the model's main insights, Proposition 2 provides the intuition behind the mechanisms at play. The simplest sensitivity to interpret is the response to changes in the total amount of T-bills. In effect, since only households hold T-bills, an increase in supply reduces their need to satisfy their demand for safe assets. The response is a reduction in repo issuance proportional to v'', scaled by the marginal increase in safe assets and their value, captured by $(1 + \gamma\kappa)$.

The model's main result is the sensitivity of repo issuance to changes in convenience yield, captured through η. Proposition 2 shows that an increase in convenience yield implies a decrease in repo issuance. The partial derivative has two parts. The first is similar to repo's sensitivity to changes T-bills outstanding: households' have a higher payoff through a mechanical increase in v, captured by v_η, reducing their demand for safe assets proportional to v''. The second effect stems from an increase in the marginal preference for safe assets times households' valuation of consuming in $t = 1$ relative to consuming in $t = 0$.[13] [14] This result goes against the conventional wisdom on how private safe asset issuance changes with the demand for safe assets. Since the model assumes that risky assets themselves have a safe asset benefit, an increase in convenience yield allocates more of them to households, reducing the amount of safe asset intermediation.[15]

The response to an increase in Treasury bonds outstanding has two opposing effects. On the one hand, the bank increases its Treasury holdings which need to be funded, implying an increase in repo supply. On the other hand, households' increase their safe asset holdings, suggesting a decrease in repo demand. The first effect is captured, in part, by the bank absorbing more volatility captured by,

$$\hat{V} - \left(\frac{\mu}{R_T} - 1\right)^2 (1 + v')R_T > 0$$

which can be interpreted as a variance risk premia.[16] In addition, households' increase in Treasury holdings reduces the benefits from having safe assets because on aggregate the portfolio becomes more volatile, captured by $\gamma\hat{V}\kappa$, also prompting an increase in repo issuance. The second effect is captured by $\left(\frac{\mu}{R_T} - 1\right)\left(\frac{\mu}{R_T} + \gamma\kappa\right)$ which comes from an increase in households' safe asset holdings, similar to an increase in T-bills, suggesting a decrease in repo issuance.

Krishnamurthy and Vissing-Jorgensen (2015) characterize similar effects to changes in U.S. Treasury outstanding. They call these effects *bank portfolio substitution effect* and *household debt substitution effect*, and note that in general the response of private safe asset creation depends on whichever dominates. In the empirical section I show that the correlation between Treasury note

[13]Since $\gamma\kappa > 1$ and because of equation (3), $\gamma\kappa(1 + v')R_T > 1$.

[14]Note that the result still holds when $v'' = 0$.

[15]In an extension of the model, one can characterize the equilibrium when the risky asset does not have a safe asset benefit. This specification should give the traditional result: an increase in convenience yield implies an increase in privately produced safe assets claims.

[16]In effect, $\hat{V} = \mathbb{E}\left(\tilde{S}(\tilde{R} - \mu)^2\right) > \frac{1}{\mathbb{E}(\tilde{S})}\left(\mathbb{E}(\tilde{S}(\tilde{R} - \mu))\right)^2 = \left(\frac{\mu}{R_T} - 1\right)^2 (1 + v')R_T$. See the appendix for details.

issuance and aggregate repo issuance is positive, suggesting that the bank portfolio substitution effect dominates.

Finally, subsection 2.3 studies the impact of the Federal Reserve's program to issue repos directly to cash investors. Proposition 4 shows that the amount of central bank repos increase as the demand for safe assets increases. The intuition is that households cannot access the Treasuries backing the central bank's repo issuance (unmodeled). Eliminating the possibility of having a reallocation of Treasuries from the central bank to households, implies that the facility must increase in size to satisfy the demand for safe assets.

3 Empirical Analysis

The comparative static results motivates a number of empirical exercises. Following an analysis similar to Sunderam (2015), the strategy is to empirically test whether the convenience yield correlates with repo issuance. Contrary to previous results, Proposition 2 predicts that a high convenience yield (i.e., an increase in η), decreases the amount of repo issuance.

3.1 Data

I use the tri-party allocation data collected by the Federal Reserve Bank of New York (FRBNY) and used internally at the Federal Reserve Board (Board). The data is collected from the two tri-party clearing banks and contains the total amount borrowed by each dealer in the tri-party system, per collateral class, at a daily frequency. From these data I can calculate changes in the amount of repo outstanding at the borrower level. The analysis uses primary dealers' repo issuance and the Federal Reserve's ON and Term repo issuance. The period of analysis is between 2nd of January 2009 and 25th of March 2016.

Following a specification similar to Sunderam (2015), and the insight from equation (13), the convenience yield for holding safe assets can be measured by the difference in returns between holding a risk free safe asset (T-bills) and a contract with a risk free payoff that does not imply physical ownership of an asset. In the data, I use the 4 week T-bill rate for the safe asset rate, downloaded from the Federal Reserve H.15 Statistical Release, and the 1 month overnight indexed swap rate (OIS) for the risk free rate, downloaded from Bloomberg.

Proposition 2 also has predictions on how changes in the total amount of T-bills and Treasury Notes and Bonds available to the public affect repo issuance. For this I use aggregate series of Treasury securities outstanding published by TreasuryDirect.[17] Given that the model has predictions on short term T-bill issuance, in some specifications I consider the total amount of T-bills outstanding with one month left to mature.

For the time series analysis I consider both weekly and daily frequency. In the panel analysis I only consider daily frequency.

Figure 1 shows an interesting pattern of repo volumes on quarter-end (also see Figure 2). Munyan (2015) shows that some firms choose to reduce the total amount of their repo outstanding to "window dress" the size of their balance sheet. This behavior is well known, and the incentive to window dress it outside the scope of this paper. Therefore, I exclude weekly and daily changes that include a quarter-end observation. Since the item of interest is to understand how repo issuance is affected by the convenience yield, for any given quarter-end week/date t, I exclude $\{t, t+1\}$. In case lagged repo issuance is used as a control variable I also have to exclude dates $\{t+2, t+3, \cdots, N_{lag}\}$, where N_{lag} is the number of lags. Note that given the exclusion criteria, more lags implies reducing the sample size. Table 1 presents summary statistics of the data, excluding quarter-end observations.

3.2 Time Series Model

The theoretical model motivates an empirical specification which considers private and public U.S. Treasury repo outstanding. I consider two distinct subsets of repo issuers: all dealers which were

[17] In a previous version of the paper I subtracted the amount of Treasuries held in the Federal Reserve's SOMA portfolio to measure the effective amount of bonds held by the public. Given that publicly available data on SOMA holdings are at a weekly frequency, I decided to show results that do not exclude SOMA holdings. The results economically and statistically similar.

once primary dealers in my sample (called 'Primary Dealers'), and the Federal Reserve's RRP program. This leads to the following specification,

$$\Delta log(RepoOut_t) = \alpha + \beta(TBill - OIS)_{t-1} + \epsilon_t \tag{16}$$

where $RepoOut_t$ is the total repo outstanding at time t for the subset of borrowers under consideration, $(TBill - OIS)_{t-1}$ is the convenience yield measured in $t-1$, and Δ is the first difference operator. That is, changes in repo outstanding—a proxy for repo issuance—regressed against lagged measures of the safe asset convenience yield. For robustness, some specifications will control for lagged repo issuance: two lags at a weekly frequency, four lags at a daily frequency.

$$\Delta log(RepoOut_t) = \alpha + \beta(TBill - OIS)_{t-1} + \sum_j \gamma_j \Delta log(RepoOut_{t-j}) + \epsilon_t \tag{17}$$

Taking the model literally, equation (13) suggests that the appropriate measure of convenience yield should be $(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$, which I call *discounted convenience yield*. Using the discounted convenience yield discounts future safe asset benefits at the safe asset rate. Given the low level of interest rates during my sample period, this modification does not lead to large differences, but in a higher rate environment, this adjustment can be substantial (see Figure 3).[18] The discounted convenience yield leads to the following specification:

$$\Delta log(RepoOut_t) = \alpha + \beta \frac{(TBill - OIS)_{t-1}}{(TBill + 1)_{t-1}} + \sum_j \gamma_j \Delta log(RepoOut_{t-j}) + \epsilon_t. \tag{18}$$

In half of the specifications I control for month-year fixed effects. The results from regressions (17) and (18) at weekly and daily frequency can be seen in Tables 2 and 3, respectively. Regressions without month-year fixed effects use Newey-West errors with 12 weeks lag at the weekly frequency, and 21 days lag at the daily frequency. Regressions with month-year fixed effects use clustered

[18]The repo data available to the Board starts in January 2009.

errors.

In the model, changes in the supply of public safe assets has an effect on repo issuance. Specifications which takes into account changes in the amount of U.S. Treasuries outstanding lead to the following regression:

$$\begin{aligned} \Delta log(RepoOut_t) &= \alpha + \beta(TBill - OIS)_{t-1} + \\ &\sum_j \gamma_j \Delta log(RepoOut_{t-j}) + \\ &+\theta_1 \Delta log(TBillsOut_t) + \theta_2 \Delta log(USTNotesOut_t) + \epsilon_t \end{aligned} \tag{19}$$

where $\Delta log(TBillsOut_t)$ and $\Delta log(USTNotesOut_t)$ are changes in the total outstanding of T-bills and U.S. Treasury notes held by the public. I also consider a specification which replaces changes in T-bill outstanding with changes in short term T-bills outstanding (i.e., T-bills with a maturity less than one month), denoted by $\Delta log(ShTBillsOut_t)$. Regression (19) is estimated using both the standard and discounted convenience yield.

Table 4 and 5 show the results of model (19) at a weekly and daily frequency, respectively. Regressions without month-year fixed effects use Newey-West errors with 12 weeks lag at the weekly frequency, and 21 days lag at the daily frequency. Regressions with month-year fixed effects use clustered errors.

The analysis of the Federal Reserve's RRP program starts on the 23-December-2013, when the cap on individual investor participation was raised from 1 billion to 3 billion, increasing the program's usage and significance. Table 6 presents results for the RRP program, and for comparison, Primary Dealer issuance over the same sample period.

3.3 Panel Model

The previous analysis can be repeated taking advantage of each individual borrower's activity. That is model (16) takes the following form,

$$\Delta log(RepoOut_{it}) = \alpha + \delta_i + \beta \frac{(TBill - OIS)_{t-1}}{(TBill + 1)_{t-1}} + \gamma X_t + \epsilon_{it}. \tag{20}$$

Where δ_i are dealer fixed effects and X_t are the same controls used in the time series specification. As before, this analysis only considers dealers which were once a primary dealer in the sample, and is run at a daily frequency with and without month-year fixed effects. Standard errors are clustered at the dealer level, and also at the month-year level when using time fixed effects (i.e., double clustering).

Given the different model implied sensitivities, I make a distinction between primary dealers and the Federal Reserve's RRP program. This leads to the following specification,

$$\Delta log(RepoOut_{it}) = \alpha + \delta_i + \beta \frac{(TBill - OIS)_{t-1}}{(TBill + 1)_{t-1}} + \gamma X_t + \beta_{FRB} 1_{FRB} (\frac{(TBill - OIS)_{t-1}}{(TBill + 1)_{t-1}} + \epsilon_{it} \tag{21}$$

where 1_{FRB} is an indicator function whenever $i = FRB$. That is, I incorporate an interaction term on the convenience yield whenever the borrower is the Federal Reserve.

3.4 Discussion of Empirical Results

At a weekly frequency, table 2 shows that when controlled for lag issuance and month-year fixed effects, the sensitivity of primary dealer's repo issuance to the convenience yield is positive and statistically significant. At a daily frequency, table 3 shows that under all specifications, the sensitivity of primary dealer's repo issuance to the convenience yield is positive and statistically significant at a significance level of 1%. Since the convenience yield is negative, these results shows that a lower convenience yield, that is a higher demand for safe assets, *reduces* the total amount of repo issuance both at a weekly and daily frequency, consistent with Proposition 2. The effect is present using both the standard and discounted measure of convenience yield, with the latter giving marginally stronger effects.

Tables 4 and 5 show a positive correlation between primary dealer repo issuance and U.S Treasury note issuance, suggesting that the bank portfolio substitution effect described in Krishnamurthy and Vissing-Jorgensen (2015) is stronger, both at a weekly and daily frequency.[19] The loading on T-bill issuance is contrary to the one prescribed by Proposition 2. This is likely due to the fact that T-bill issuance volumes are calculated using all T-bill outstanding, irrespective of maturity (note the small loading relative to U.S. Treasury notes). The theoretical results correspond to short maturity bills, calling for a finer partition of T-bill issuance. The results from including short term T-bill issuance are not statistically significant at a weekly frequency, but their inclusion does increase the significance of both the standard and discounted convenience yield. Short term T-bill issuance does have a strong effect at a daily frequency, in line with the model's prediction: more short term public safe assets reduces repo issuance.

Importantly, the effect of both the standard and discounted convenience yield on primary dealer issuance when controlling for Treasury issuance still holds at a weekly and daily frequency, providing further evidence for the model's main intuition.

Table 6 focuses on the time when the Federal Reserve's RRP program becomes relevant. The response of private repo issuance is in line with tables 4 and 5, but the sensitivity of the Federal Reserve's operations are the complete opposite, as suggested by Proposition 4. That is, whenever the marginal convenience yield is high, takeup at the Federal Reserve's repo facility increases, suggesting that the Federal Reserve does provide safe assets to the economy when needed, in line with Carlson et al. (2014).

Finally, the results from the panel analysis in Table 7 further confirm the results. In both specifications, that exclude and include the Federal Reserve's RRP program, the principal loading on the convenience yield is positive and statistically significant. The sensitivity to U.S. Treasury

[19]Nagel (2016) also suggests that in the short run, changes in public asset safe asset supply would be absorbed by intermediaries, resulting in an increase in privately produced safe assets.

note issuance and short term T-bills issuance is also confirmed. Also, in line with Table 6, the interaction term capturing the Federal Reserve's specific sensitivity (see equation (21)), goes in the opposite direction and is statistically significant. These results suggest that the Federal Reserve's program acts as a substitute to privately issued repos whenever the demand for safe assets is high.

4 Concluding Remarks

This paper provides evidence that the private sector's ability to satisfy the demand for safe assets depends on the type of liabilities it creates. Contrary to standard results, if the underlying collateral which backs privately produced safe assets is a public safe asset, then the demand for safe assets can be satisfied by *reducing* private safe asset creation. This paper argues that the creation of private safe assets with long term public safe assets is beneficial because it shifts risk to agents which have a higher tolerance for it.

This insight can have implications on liquidity regulation and wholesale funding in general. The model suggests that the banking sector can provide an important service by assuming the risk of government debt and issuing short term claims to satisfy the demand for safe assets. But this operation does not create safe assets, it merely transforms them. The model suggests that private safe asset creation occurs when the assets backing them do not already have a safe asset premium.

In addition, the results suggest that the Federal Reserve can expand the public supply of safe assets with its RRP program. By circumventing the banking sector and issuing liabilities directly to cash investors, the Federal Reserve can satisfy the demand for safe assets without relying on the traditional banking sector or altering its portfolio holdings. This effect can have important macro prudential implications.

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Figure 1: Total Tri-Party Repo Outstanding Backed by U.S. Treasury Collateral
The three series depict total primary dealer volumes, other dealer volumes, and the Federal Reserves ON & Term RRP program separately.

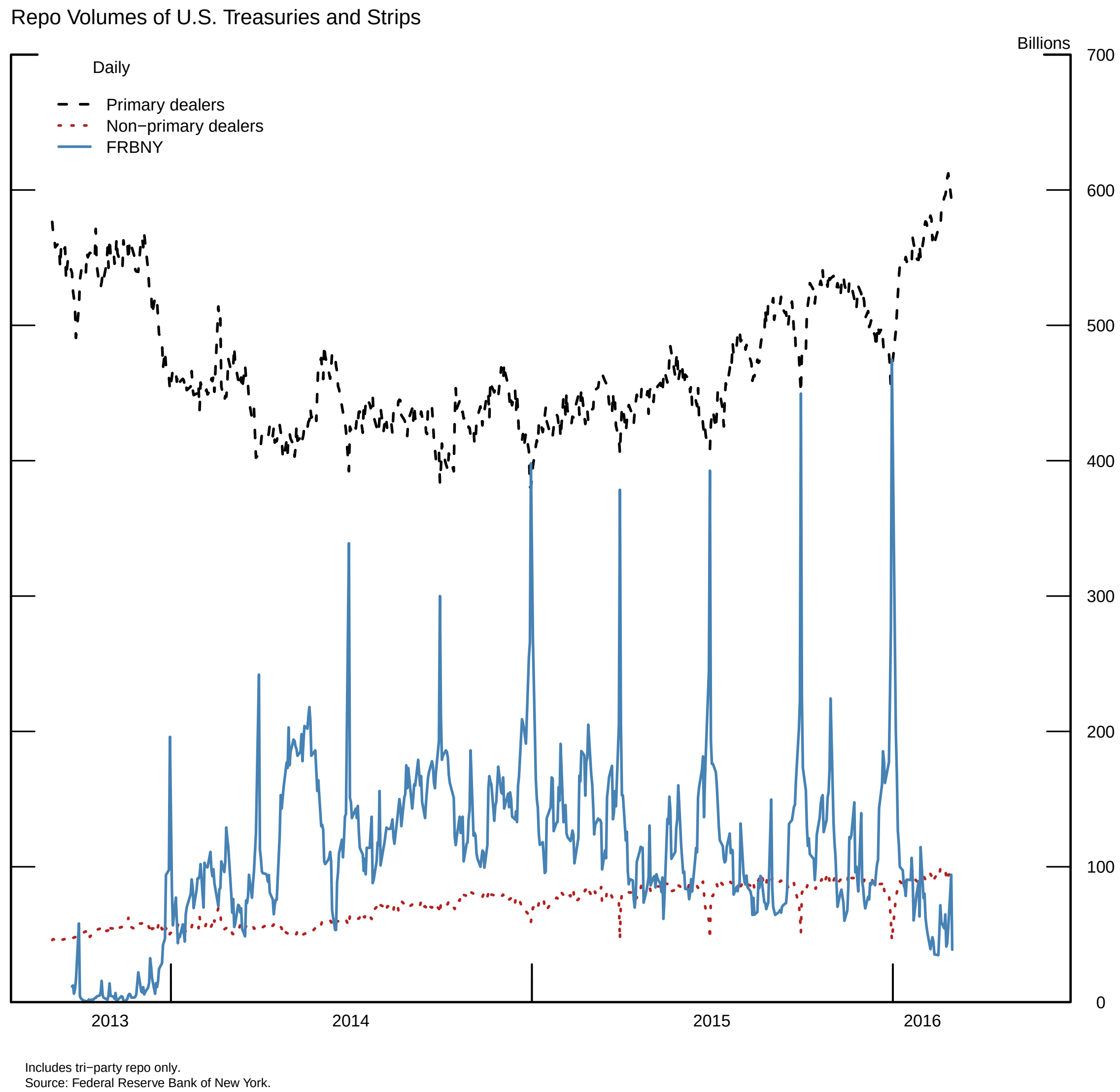

Figure 2: Total Private Repo Issuance
The series depicts daily changes in Primary Dealer repo outstanding. Changes in outstanding around quarter end are highlighted in red dots. These observations are excluded from the empirical analysis.

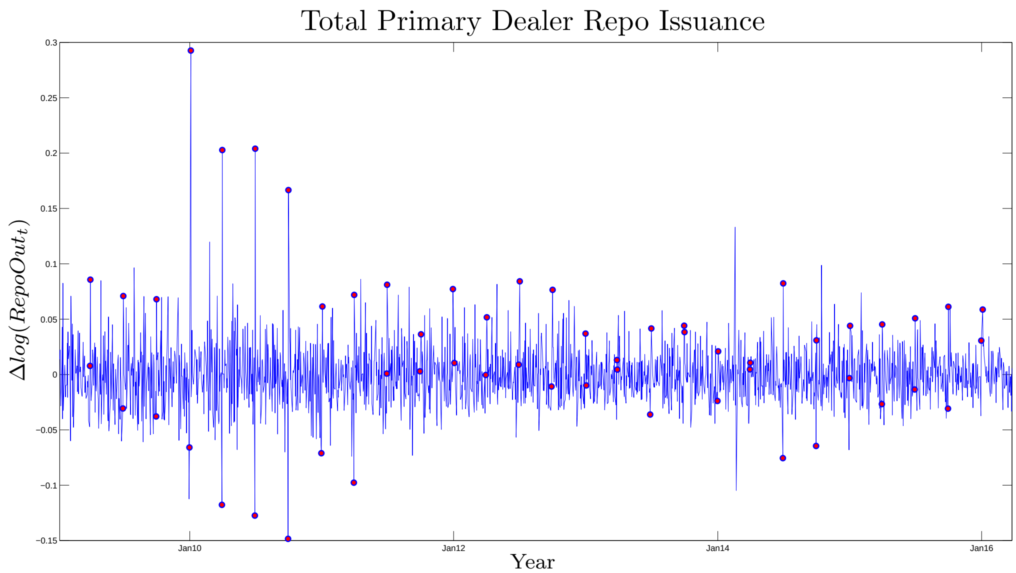

Figure 3: Convenience Yield vs Discounted Convenience Yield
The series depicts the daily convenience yield, measured as the difference between the 4 week T-bill rate and the one month OIS rate, and the discounted convenience yield, measured by the convenience yield divided by the gross return of the 4 week T-bill. The dashed line indicates the start of the empirical analysis in January 2009.

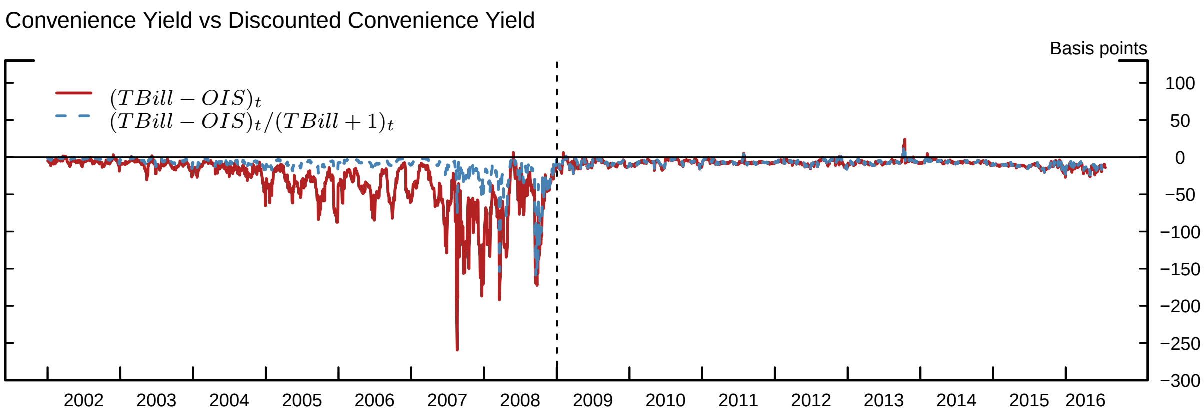

Table 1: Summary Statistics

This table presents summary statistics for the variables used in the paper. $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers. $(TBill - OIS)_{t-1}$ is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate. $(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. $\Delta log(TBillsOut_t)$ is the log change in Treasury bills outstanding accessible to the public, $\Delta log(ShTBillsOut_t)$ is the log change in Treasury bills outstanding with maturity less than one month accessible to the public, and $\Delta log(USTNotesOut_t)$ is the log change in U.S. Treasury notes outstanding accessible to the public. Panel A shows statistics for weekly data and Panel B shows statistics for daily data. The sample runs from January 2009 till March 2016. Quarter end dates are excluded.

Panel A: Weekly Data

	Count	Mean	Sdev	Min	Max
PD $\Delta log(RepoOut_t)$	366	-.0000	.0454	-.1309	.1953
$(TBill - OIS)_{t-1}$	366	-.07391	.0389	-.2705	.1540
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$	366	-.0700	.0373	-.2504	.1213
$\Delta log(TBillsOut_t)$	366	-.0000	.0137	-.0605	.0504
$\Delta log(ShTBillsOut_t)$	366	.0003	.0719	-.2740	.2856
$\Delta log(USTNotesOut_t)$	366	.0029	.0048	-.0016	.0219

Panel B: Daily Data

	Count	Mean	Sdev	Min	Max
PD $\Delta log(RepoOut_t)$	1,759	-.0007	.0244	-.1125	.1332
FRB $\Delta log(RepoOut_t)$	547	.0012	.1780	-.8871	.7107
$(TBill - OIS)_{t-1}$	1,809	-.0783	.0398	-.2705	.2430
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$	1,809	-.0744	.0379	-.2504	.1800
$\Delta log(TBillsOut_t)$	1,787	-.0001	.0068	-.0805	.0805
$\Delta log(ShTBillsOut_t)$	1,787	-.0001	.0998	-.3480	.4875
$\Delta log(USTNotesOut_t)$	1,787	.0006	.0026	-.0022	.0219

Table 2: Weekly Primary Dealer Repo Issuance
This table shows regressions of the following form:

$$\Delta log(RepoOut_t) = \alpha + \beta CY_{t-1} + \epsilon_t$$

Where the $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers. CY_{t-1} is the convenience yield measured by either $(TBill - OIS)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate or $(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. $\Delta log(RepoOut_{t-j})$ is the j-th lagged log change of repo outstanding. The sample runs weekly from January 2009 till March 2016. Quarter end dates are excluded. Regressions run with and without month-year fixed effects. Newey-West t-statistics with 12 lags are reported in regressions without fixed effects (NW), clustered errors t-statistics are reported in regressions with fixed effects (SC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_t)$ of Primary Dealer Repo Volume							
Intercept	0.002	0.031	0.005	0.039*	0.002	0.031	0.005	0.042*
	(0.454)	(1.627)	(0.817)	(1.681)	(0.476)	(1.593)	(0.880)	(1.739)
$(TBill - OIS)_{t-1}$	0.034	0.143	0.064	0.299**				
	(0.497)	(0.980)	(0.840)	(2.194)				
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$					0.037	0.151	0.071	0.323**
					(0.525)	(0.963)	(0.918)	(2.231)
$\Delta log(RepoOut_{t-1})$			-0.195***	-0.399***			-0.196***	-0.403***
			(-3.768)	(-5.964)			(-3.784)	(-6.053)
$\Delta log(RepoOut_{t-2})$			-0.052	-0.183***			-0.052	-0.185***
			(-1.179)	(-3.368)			(-1.190)	(-3.406)
R^2	0.001	0.218	0.037	0.345	0.001	0.218	0.037	0.346
N obs	360	355	354	349	360	355	354	349
Month-Year FE	N	Y	N	Y	N	Y	N	Y
Error Type	NW	SC	NW	SC	NW	SC	NW	SC

Table 3: Daily Primary Dealer Repo Issuance
This table shows regressions of the following form:

$$\Delta log(RepoOut_t) = \alpha + \beta CY_{t-1} + \epsilon_t$$

Where the $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers. CY_{t-1} is the convenience yield measured by either $(TBill - OIS)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate or $(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. $\Delta log(RepoOut_{t-j})$ is the j-th lagged log change of repo outstanding. The sample runs daily from January 2009 till March 2016. Quarter end dates are excluded. Regressions run with and without month-year fixed effects. Newey-West t-statistics with 21 lags are reported in regressions without fixed effects (NW), clustered errors t-statistics are reported in regressions with fixed effects (SC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_t)$ of Primary Dealer Repo Volume							
Intercept	0.003***	0.015***	0.004***	0.024***	0.003***	0.016***	0.004***	0.025***
	(3.038)	(3.839)	(3.085)	(4.693)	(3.275)	(4.152)	(3.330)	(5.298)
$(TBill - OIS)_{t-1}$	0.048***	0.075***	0.070***	0.123***				
	(3.593)	(2.724)	(4.135)	(3.544)				
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$					0.052***	0.083***	0.076***	0.136***
					(3.827)	(3.000)	(4.397)	(4.060)
$\Delta log(RepoOut_{t-1})$			-0.229***	-0.286***			-0.230***	-0.287***
			(-9.828)	(-11.481)			(-9.845)	(-11.558)
$\Delta log(RepoOut_{t-2})$			-0.077***	-0.135***			-0.078***	-0.136***
			(-3.197)	(-4.737)			(-3.234)	(-4.777)
$\Delta log(RepoOut_{t-3})$			-0.035	-0.089***			-0.036	-0.090***
			(-1.176)	(-2.802)			(-1.190)	(-2.834)
$\Delta log(RepoOut_{t-4})$			-0.068***	-0.113***			-0.069***	-0.114***
			(-2.743)	(-4.550)			(-2.771)	(-4.616)
R^2	0.006	0.050	0.061	0.131	0.006	0.051	0.062	0.133
N obs	1751	1751	1634	1634	1751	1751	1634	1634
Month-Year FE	N	Y	N	Y	N	Y	N	Y
Error Type	NW	SC	NW	SC	NW	SC	NW	SC

Table 4: Weekly Primary Dealer Repo Issuance with Treasury Issuance Controls
This table shows regressions of the following form:

$$\Delta log(RepoOut_t) = \alpha + \beta CY_{t-1} + \theta\Delta(TreasuryIssuance)_t + \epsilon_t$$

Where the $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers. CY_{t-1} is the convenience yield measured by either $(TBill - OIS)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate or $(TBill - OIS)_{t-1}/(TBill+1)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. 2 lags of $\Delta log(RepoOut_t)$ are included as controls (not shown). $\Delta log(TBillsOut_t)$ is the log change in Treasury bills outstanding accessible to the public, $\Delta log(ShTBillsOut_t)$ is the log change in Treasury bills outstanding with maturity less than one month accessible to the public, and $\Delta log(USTNotesOut_t)$ is the log change in U.S. Treasury notes outstanding accessible to the public. The sample runs weekly from January 2009 till March 2016. Quarter end dates are excluded. Regressions run with and without month-year fixed effects. Newey-West t-statistics with 12 lags are reported in regressions without fixed effects (NW), clustered errors t-statistics are reported in regressions with fixed effects (SC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_t)$ of Primary Dealer Repo Volume							
Intercept	-0.007	0.029	-0.005	0.036	-0.007	0.032	-0.005	0.039*
	(-1.289)	(1.371)	(-0.841)	(1.620)	(-1.357)	(1.466)	(-0.869)	(1.743)
$(TBill - OIS)_{t-1}$	0.027	0.257**	0.059	0.308**				
	(0.339)	(2.022)	(0.694)	(2.319)				
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$					0.027	0.281**	0.062	0.336**
					(0.324)	(2.094)	(0.733)	(2.421)
$\Delta log(TBillsOut_t)$	0.391***	0.558**			0.391***	0.548**		
	(2.897)	(2.202)			(2.871)	(2.150)		
$\Delta log(ShTBillsOut_t)$			0.018	0.014			0.017	0.014
			(0.546)	(0.317)			(0.536)	(0.310)
$\Delta log(USTNotesOut_t)$	3.197***	3.689***	3.150***	3.687***	3.196***	3.699***	3.147***	3.699***
	(4.370)	(5.390)	(4.346)	(5.484)	(4.365)	(5.422)	(4.332)	(5.522)
R^2	0.151	0.463	0.139	0.449	0.151	0.465	0.139	0.451
N obs	354	349	354	349	354	349	354	349
Month-Year FE	N	Y	N	Y	N	Y	N	Y
Error Type	NW	SC	NW	SC	NW	SC	NW	SC

Table 5: Daily Primary Dealer Repo Issuance with Treasury Issuance Control
This table shows regressions of the following form:

$$\Delta log(RepoOut_t) = \alpha + \beta CY_{t-1} + \theta\Delta(TreasuryIssuance)_t + \epsilon_t$$

Where the $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers. CY_{t-1} is the convenience yield measured by either $(TBill - OIS)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate or $(TBill-OIS)_{t-1}/(TBill+1)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. 4 lags of $\Delta log(RepoOut_t)$ are included as controls (not shown). $\Delta log(TBillsOut_t)$ is the log change in Treasury bills outstanding accessible to the public, $\Delta log(ShTBillsOut_t)$ is the log change in Treasury bills outstanding with maturity less than one month accessible to the public, and $\Delta log(USTNotesOut_t)$ is the log change in U.S. Treasury notes outstanding accessible to the public. The sample runs daily from January 2009 till March 2016. Quarter end dates are excluded. Regressions run with and without month-year fixed effects. Newey-West t-statistics with 21 lags are reported in regressions without fixed effects (NW), clustered errors t-statistics are reported in regressions with fixed effects (SC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_t)$ of Primary Dealer Repo Volume							
Intercept	0.001	0.020***	0.002*	0.022***	0.001	0.022***	0.002**	0.023***
	(1.027)	(4.811)	(1.828)	(4.861)	(1.098)	(5.499)	(2.014)	(5.662)
$(TBill - OIS)_{t-1}$	0.059***	0.113***	0.072***	0.128***				
	(3.623)	(3.858)	(4.309)	(4.180)				
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$					0.063***	0.125***	0.077***	0.141***
					(3.760)	(4.465)	(4.601)	(4.913)
$\Delta log(TBillsOut_t)$	0.423***	0.419***			0.418***	0.414***		
	(3.874)	(3.663)			(3.836)	(3.640)		
$\Delta log(ShTBillsOut_t)$			-0.053***	-0.051***			-0.053***	-0.051***
			(-9.447)	(-9.367)			(-9.474)	(-9.389)
$\Delta log(USTNotesOut_t)$	3.123***	3.166***	3.012***	3.061***	3.117***	3.164***	3.006***	3.059***
	(10.813)	(11.278)	(10.848)	(11.119)	(10.776)	(11.243)	(10.802)	(11.080)
R^2	0.159	0.227	0.188	0.255	0.159	0.228	0.189	0.256
N obs	1615	1615	1615	1615	1615	1615	1615	1615
Month-Year FE	N	Y	N	Y	N	Y	N	Y
Error Type	NW	SC	NW	SC	NW	SC	NW	SC

Table 6: Daily Primary Dealer and Federal Reserve Repo Issuance with Treasury Issuance Control
This table shows regressions of the following form:

$$\Delta log(RepoOut_t) = \alpha + \beta CY_{t-1} + \theta\Delta(TreasuryIssuance)_t + \epsilon_t$$

Where the $\Delta log(RepoOut_t)$ is the log change of total repo outstanding by primary dealers and the Federal Reserve's ON & Term RRP program. CY_{t-1} is the convenience yield measured by either $(TBill - OIS)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week overnight index swap (OIS) rate or $(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$ which is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. 4 lags of $\Delta log(RepoOut_t)$ are included as controls (not shown). $\Delta log(ShTBillsOut_t)$ is the log change in Treasury bills outstanding with maturity less than one month accessible to the public and $\Delta log(USTNotesOut_t)$ is the log change in U.S. Treasury notes outstanding accessible to the public. The sample runs daily from December 23rd, 2013 till March 2016. Quarter end dates are excluded. Regressions run with and without month-year fixed effects. Newey-West t-statistics with 21 lags are reported in regressions without fixed effects (NW), clustered errors t-statistics are reported in regressions with fixed effects (SC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_t)$							
	Primary Dealers				FRB			
Intercept	0.001	0.001	0.001	0.001	-0.021	0.014	-0.023	0.016
	(0.300)	(0.381)	(0.337)	(0.544)	(-1.160)	(0.579)	(-1.288)	(0.607)
$(TBill - OIS)_{t-1}$	0.033	0.103**			-0.343*	-0.914**		
	(1.294)	(2.537)			(-1.798)	(-2.289)		
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$			0.036	0.113**			-0.387*	-0.904**
			(1.341)	(2.516)			(-1.941)	(-2.145)
$\Delta log(ShTBillsOut_t)$	-0.032***	-0.032***	-0.033***	-0.032***	0.032	0.048	0.034	0.049
	(-3.426)	(-3.242)	(-3.429)	(-3.270)	(0.425)	(0.582)	(0.449)	(0.587)
$\Delta log(USTNotesOut_t)$	7.126***	7.418***	7.111***	7.401***	-14.099	-17.301	-13.953	-17.098
	(3.406)	(3.746)	(3.393)	(3.735)	(-1.109)	(-1.547)	(-1.096)	(-1.524)
R^2	0.101	0.171	0.101	0.172	0.043	0.073	0.043	0.072
N obs	497	497	497	497	494	494	494	494
Month-Year FE	N	Y	N	Y	N	Y	N	Y
Error Type	NW	SC	NW	SC	NW	SC	NW	SC

Table 7: Panel of Primary Dealer and Federal Reserve Repo Issuance with Treasury Issuance Control
This table shows regressions of the following form:

$$\Delta log(RepoOut_{it}) = \alpha + \delta_i + \beta(TBill - OIS)_{t-1}/(TBill + 1)_{t-1} + \theta\Delta(TreasuryIssuance)_t + \epsilon_{it}$$

Where the $\Delta log(RepoOut_{it})$ is the log change of repo outstanding of primary dealer i and and the Federal Reserves ON & Term RRP program. $(TBill - OIS)_{t-1}/(TBill+1)_{t-1}$ is the spread of the four-week Treasury bills over the four-week OIS, divided by the gross rate of the four-week Treasury bill. Some specifications contain 4 lags of $\Delta log(RepoOut_{it})$ as controls which are indicated in '4 LHS Lag' row (not shown). $\Delta log(ShTBillsOut_t)$ is the log change in Treasury bills outstanding with maturity less than one month accessible to the public and $\Delta log(USTNotesOut_t)$ is the log changes in U.S. Treasury notes outstanding accessible to the public. 1_{FRB} is an indicator dummy equal to 1 whenever $i = FRB$. The sample runs weekly from January 2009 till March 2016. Quarter end dates are excluded. Regressions run with dealers and month-year fixed effects. Single clustered errors t-statistics around dealer fixed effects (SC), double clustered t-statistics around dealer and month-year fixed effects (DC). *, **, ***, denote significance at the 10%, 5%, and 1% levels, respectively.

	LHS: $\Delta log(RepoOut_{it})$								
	Only Primary Dealers				Primary Dealers & FRB				
Intercept	0.004	0.019	0.025***	0.019***	0.004	0.018	0.022**	0.015**	0.019***
	(1.657)	(1.602)	(2.603)	(2.891)	(1.588)	(1.527)	(2.183)	(2.189)	(2.900)
$(TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$	0.070**	0.147**	0.174***	0.187***	0.068**	0.136*	0.149***	0.161***	0.186***
	(2.167)	(1.997)	(3.165)	(3.325)	(2.105)	(1.921)	(2.612)	(2.749)	(3.324)
$\Delta log(ShTBillsOut_t)$				-0.070***				-0.069***	-0.069***
				(-4.045)				(-4.005)	(-4.008)
$\Delta log(USTNotesOut_t)$				2.716***				2.687***	2.683***
				(3.472)				(3.458)	(3.451)
$1_{FRB} \times (TBill - OIS)_{t-1}/(TBill + 1)_{t-1}$									-0.671***
									(-9.509)
R^2	0.000	0.001	0.236	0.239	0.000	0.001	0.232	0.236	0.236
N obs	35404	35404	32970	32586	35949	35949	33477	33080	33080
4 LHS Lags	N	N	Y	Y	N	N	Y	Y	Y
Dealer FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Month-Year FE	N	Y	Y	Y	N	Y	Y	Y	Y
Error Type	SC	DC	DC	DC	SC	DC	DC	DC	DC

A Appendix

Proof of Proposition 2:

The result is derived from invoking the Implicit Function Theorem. Consider the two equilibrium equations,

$$T_1 = \mathbb{E}(\tilde{S})(1 + v'(M;\eta))R_T - 1 = 0$$
$$T_2 = \mathbb{E}(\tilde{S}(\tilde{R} - \mu)) + \mathbb{E}(\tilde{S})(1 + v'(M;\eta))\mu - 1 = 0$$

where T_1 and T_2 stems from rewriting equation (3) and (12). Recall, that the equilibrium variables are R_T and μ and given the Proposition's cost function is $RP = \frac{1}{c}(\mu - R_T)$. Thus the result depends on studying the sensitivities of $\mu - R_T$ with respect to model parameters.

$$\begin{pmatrix} \frac{\partial R_T}{\partial x} \\ \frac{\partial \mu}{\partial x} \end{pmatrix} = -\underbrace{\begin{bmatrix} \frac{\partial T_1}{\partial R_T} & \frac{\partial T_1}{\partial \mu} \\ \frac{\partial T_2}{\partial R_T} & \frac{\partial T_2}{\partial \mu} \end{bmatrix}^{-1}}_{:=D^{-1}} \begin{pmatrix} \frac{\partial T_1}{\partial x} \\ \frac{\partial T_2}{\partial x} \end{pmatrix}$$

where $x \in \{\eta, \Theta_T, \Theta_U\}$. To simplify the derivation, define $g := \mathbb{E}(\tilde{S})(1 + v'(M;\eta))$ and $h := \mathbb{E}(\tilde{R}(\tilde{S} - \mu))$, therefore,

$$\frac{\partial T_1}{\partial R_T} = g + \frac{\partial g}{\partial R_T} R_T$$
$$\frac{\partial T_1}{\partial \mu} = \frac{\partial g}{\partial \mu} R_T$$
$$\frac{\partial T_2}{\partial R_T} = \frac{\partial h}{\partial R_T} + \frac{\partial g}{\partial R_T}\mu$$
$$\frac{\partial T_2}{\partial \mu} = \frac{\partial h}{\partial \mu} + \frac{\partial g}{\partial \mu}\mu + g$$

First, I calculate the inverse of D, which has the following determinant,

$$\begin{aligned} |D| &= \frac{\partial T_1}{\partial R_T}\frac{\partial T_2}{\partial \mu} - \frac{\partial T_1}{\partial \mu}\frac{\partial T_2}{\partial R_T} \\ &= \left(\frac{\partial g}{\partial R_T}\frac{\partial h}{\partial \mu} - \frac{\partial g}{\partial \mu}\frac{\partial h}{\partial R_T}\right) R_T + \left(\frac{\partial g}{\partial \mu}\mu + \frac{\partial g}{\partial R_T} R_T\right) g + \frac{\partial h}{\partial \mu} g + g^2 \end{aligned} \tag{22}$$

The equilibrium in Proposition 1 results in the following household consumption and money holdings,

$$\begin{aligned} c_0 &= e_0 - \Theta_T - (\Theta_U - w) \\ M &= R_T\Theta_T + \mu(\Theta_U - w) - (\mu - R_T)^2\frac{1}{c} \\ \tilde{c}_1 &= R_T\Theta_T + \tilde{R}(\Theta_U - w) - (\tilde{R} - R_T)(\mu - R_T)\frac{1}{c} \\ &= M + (\tilde{R} - \mu)\left[(\Theta_U - w) - (\mu - R_T)\frac{1}{c}\right] \end{aligned}$$

Defining $\tilde{S}_{c1} := \frac{\partial \tilde{S}}{\partial c_1}$ we can have simple expressions for changes in the SDF due to CARA utility's defining property: $u''(c) = -\gamma u'(c)$. This implies that $\tilde{S}_{c1} = -\gamma\tilde{S}$ and $\tilde{S}_{c0} = \gamma\tilde{S}$. Using the Euler equations gives the following partial derivatives of g and h,

$$\begin{aligned} \frac{\partial g}{\partial R_T} &= \frac{1}{c}\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu))(1 + v') + \left(\mathbb{E}(\tilde{S}_{c1})(1 + v')^2 + v''\mathbb{E}(\tilde{S})\right)\frac{\partial M}{\partial R_T} \\ &= \gamma\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)(1 + v') - \left(\gamma\frac{(1 + v')}{R_T} - v''\mathbb{E}(\tilde{S})\right)\frac{\partial M}{\partial R_T} \\ \frac{\partial g}{\partial \mu} &= -\frac{1}{c}\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu))(1 + v') + \left(\mathbb{E}(\tilde{S}_{c1})(1 + v')^2 + v''\mathbb{E}(\tilde{S})\right)\frac{\partial M}{\partial \mu} \\ &= -\gamma\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)(1 + v') - \left(\gamma\frac{(1 + v')}{R_T} - v''\mathbb{E}(\tilde{S})\right)\frac{\partial M}{\partial \mu} \\ \frac{\partial h}{\partial R_T} &= \frac{1}{c}\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu)^2) + \left(\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu))(1 + v')\right)\frac{\partial M}{\partial R_T} \\ &= -\gamma\frac{1}{c}\hat{V} + \gamma\left(\frac{\mu}{R_T} - 1\right)(1 + v')\frac{\partial M}{\partial R_T} \\ \frac{\partial h}{\partial \mu} &= -\frac{1}{c}\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu)^2) + \left(\mathbb{E}(\tilde{S}_{c1}(\tilde{R} - \mu))(1 + v')\right)\frac{\partial M}{\partial \mu} \\ &= \gamma\frac{1}{c}\hat{V} + \gamma\left(\frac{\mu}{R_T} - 1\right)(1 + v')\frac{\partial M}{\partial \mu} \end{aligned}$$

where $\hat{V} = \mathbb{E}(\tilde{S}(\tilde{R} - \mu)^2)$.

Household's optimal portfolio implies,

$$\frac{\partial \tilde{c}_1}{\partial R_T} = \frac{\partial M}{\partial R_T} + (\tilde{R} - \mu)\frac{1}{c}, \quad \frac{\partial \tilde{c}_1}{\partial \mu} = \frac{\partial M}{\partial \mu} - (\tilde{R} - \mu)\frac{1}{c}$$

with

$$\frac{\partial M}{\partial R_T} = \Theta_T + 2(\mu - R_T)\frac{1}{c}, \quad \frac{\partial M}{\partial \mu} = \Theta_U - w - 2(\mu - R_T)\frac{1}{c}.$$

Therefore,

$$\frac{\partial g}{\partial R_T}\frac{\partial h}{\partial \mu} - \frac{\partial g}{\partial \mu}\frac{\partial h}{\partial R_T} = \gamma\frac{1}{c}\hat{V}\left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right)\left(\frac{\partial M}{\partial R_T} + \frac{\partial M}{\partial \mu}\right) + \frac{1}{c}\gamma^2\left(\frac{\mu}{R_T} - 1\right)^2 (1+v')^2\left(\frac{\partial M}{\partial R_T} + \frac{\partial M}{\partial \mu}\right)$$

and

$$\frac{\partial g}{\partial \mu}\mu + \frac{\partial g}{\partial R_T}R_T = -\gamma\frac{R_T}{c}\left(\frac{\mu}{R_T} - 1\right)^2 (1+v') - \left(\frac{\gamma(1+v')}{R_T} - v''\mathbb{E}(\tilde{S})\right)\left(\frac{\partial M}{\partial \mu}\mu + \frac{\partial M}{\partial R_T}R_T\right)$$

Returning to equation (22),

$$\begin{aligned}|D| = {} & \left(\gamma\frac{1}{c}\hat{V}\left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right) + \frac{1}{c}\gamma^2\left(\frac{\mu}{R_T} - 1\right)^2 (1+v')^2\right)\left(\frac{\partial M}{\partial R_T} + \frac{\partial M}{\partial \mu}\right)R_T \\ & \left(-\gamma R_T\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)^2 (1+v') + \left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right)\left(\frac{\partial M}{\partial \mu}\mu + \frac{\partial M}{\partial R_T}R_T\right)\right)\frac{1}{R_T} \\ & \left(\gamma\frac{1}{c}\hat{V} + \gamma\left(\frac{\mu}{R_T} - 1\right)(1+v')\frac{\partial M}{\partial \mu}\right)\frac{1}{R_T} + \frac{1}{R_T^2}\end{aligned}$$

Note that $\frac{\partial M}{\partial \mu}\mu + \frac{\partial M}{\partial R_T}R_T = \frac{\partial M}{\partial \mu}(\mu - R_T) + (\frac{\partial M}{\partial \mu} + \frac{\partial M}{\partial R_T})R_T$, and denoting $(\frac{\partial M}{\partial \mu} + \frac{\partial M}{\partial R_T}) = \Theta_T + \Theta_U - w := \kappa$ gives,

$$\begin{aligned}|D| = {} & \left(\gamma\frac{1}{c}\hat{V}\left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right) + \frac{1}{c}\gamma^2\left(\frac{\mu}{R_T} - 1\right)^2 (1+v')^2\right)\kappa R_T \\ & -\gamma\frac{1}{c}\left(\frac{\mu}{R_T} - 1\right)^2 (1+v') + \left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right)\kappa + \left(-\frac{\gamma(1+v')}{R_T} + v''\mathbb{E}(\tilde{S})\right)\frac{\partial M}{\partial \mu}\left(\frac{\mu}{R_T} - 1\right) \\ & \left(\gamma\frac{1}{c}\hat{V} + \gamma\left(\frac{\mu}{R_T} - 1\right)(1+v')\frac{\partial M}{\partial \mu}\right)\frac{1}{R_T} + \frac{1}{R_T^2} \\ = {} & \gamma\frac{1}{c}\left(\hat{V} - \left(\frac{\mu}{R_T} - 1\right)^2 (1+v')R_T\right)\left(-\gamma\kappa(1+v') + \frac{1}{R_T}\right) + \\ & \frac{1}{R_T}\left(-\gamma\kappa(1+v') + \frac{1}{R_T}\right) + v''\mathbb{E}(\tilde{S})\left(\gamma\kappa\frac{1}{c}\hat{V}R_T + \frac{\partial M}{\partial \mu}\left(\frac{\mu}{R_T} - 1\right) + \kappa\right).\end{aligned}$$

Note that $-\gamma\kappa(1+v') + \frac{1}{R_T} < 0$ since $\gamma\kappa > 1$ and $\frac{1}{\mathbb{E}(\tilde{S})} < 1$. Note that the following expression

$$\hat{V} - \left(\frac{\mu}{R_T} - 1\right)^2 (1+v')R_T$$

is positive. In effect,

$$\hat{V} = \mathbb{E}\left(\frac{\mathbb{E}(\tilde{S})\tilde{S}}{\mathbb{E}(\tilde{S})}(\tilde{R}-\mu)^2\right) = \mathbb{E}(\tilde{S})\hat{\mathbb{E}}\left((\tilde{R}-\mu)^2\right) > \mathbb{E}(\tilde{S})\left(\hat{\mathbb{E}}(\tilde{R}-\mu)\right)^2 = \frac{1}{\mathbb{E}(\tilde{S})}\left(\mathbb{E}(\tilde{S}(\tilde{R}-\mu))\right)^2 = \left(\frac{\mu}{R_T}-1\right)^2(1+v')R_T$$

where $\hat{\mathbb{E}}$ is the risk neutral measure. That is, deflating by the risk free rate, changing measure, and applying Jensen's inequality gives the desired bound. Therefore, since $v'' < 0$ and $\frac{\partial M}{\partial \mu}\left(\frac{\mu}{R_T}-1\right)+\kappa = \frac{\mu}{R_T}\frac{\partial M}{\partial \mu}+\frac{\partial M}{\partial R_T} > \kappa > 0$, $|D| < 0$.

Given that the interest of the proposition is the repo issuance sensitivity,

$$\frac{\partial RP}{\partial x} = \frac{1}{c}\left(\frac{\partial \mu}{\partial x} - \frac{\partial R_T}{\partial x}\right)$$

Using the expression for the implicit function theorem above, gives

$$\frac{\partial \mu}{\partial x} - \frac{\partial R_T}{\partial x} = \frac{-1}{|D|}\left\{\left(\frac{\partial T_1}{\partial R_T}+\frac{\partial T_1}{\partial \mu}\right)\frac{\partial T_2}{\partial x} - \left(\frac{\partial T_2}{\partial R_T}+\frac{\partial T_2}{\partial \mu}\right)\frac{\partial T_1}{\partial x}\right\}$$

note that from the above characterizations we have,

$$\begin{aligned}
\frac{\partial T_1}{\partial R_T}+\frac{\partial T_1}{\partial \mu} &= \frac{\partial g}{\partial R_T}R_T + \frac{\partial g}{\partial \mu}R_T + g \\
&= R_T\kappa\left(-\gamma\frac{(1+v')}{R_T}+v''\mathbb{E}(\tilde{S})\right)+\frac{1}{R_T} \\
\frac{\partial T_2}{\partial R_T}+\frac{\partial T_2}{\partial \mu} &= \frac{\partial h}{\partial R_T}+\frac{\partial h}{\partial \mu}+\frac{\partial g}{\partial R_T}\mu+\frac{\partial g}{\partial \mu}\mu+g \\
&= \gamma\kappa\left(\frac{\mu}{R_T}-1\right)(1+v')+\mu\kappa\left(-\gamma\frac{(1+v')}{R_T}+v''\mathbb{E}(\tilde{S})\right)+\frac{1}{R_T}
\end{aligned}$$

Therefore,

$$\frac{\partial \mu}{\partial x}-\frac{\partial R_T}{\partial x} = \frac{-1}{|D|}\left\{\kappa\left(-\gamma\frac{(1+v')}{R_T}+v''\mathbb{E}(\tilde{S})\right)\left(\frac{\partial T_2}{\partial x}R_T-\frac{\partial T_1}{\partial x}\mu\right)-\gamma\kappa\left(\frac{\mu}{R_T}-1\right)(1+v')\frac{\partial T_1}{\partial x}+\frac{1}{R_T}\left(\frac{\partial T_2}{\partial x}-\frac{\partial T_1}{\partial x}\right)\right\}$$

In addition, $\frac{\partial T_1}{\partial x} = \frac{\partial g}{\partial x}R_T$ and $\frac{\partial T_2}{\partial x} = \frac{\partial h}{\partial x}+\frac{\partial g}{\partial x}\mu$, therefore,

$$\begin{aligned}
\frac{\partial \mu}{\partial x}-\frac{\partial R_T}{\partial x} &= \frac{-1}{|D|}\left\{\kappa\left(-\gamma\frac{(1+v')}{R_T}+v''\mathbb{E}(\tilde{S})\right)\frac{\partial h}{\partial x}R_T-\gamma\kappa\left(\frac{\mu}{R_T}-1\right)(1+v')\frac{\partial g}{\partial x}R_T+\frac{1}{R_T}\left(\frac{\partial g}{\partial x}(\mu-R_T)+\frac{\partial h}{\partial x}\right)\right\} \\
& \frac{-1}{|D|}\left\{\left(-\kappa\gamma(1+v')+\frac{1}{R_T}\right)\frac{\partial h}{\partial x}+\kappa v''\mathbb{E}(\tilde{S})R_T\frac{\partial h}{\partial x}+\left(\frac{\mu}{R_T}-1\right)R_T\left(-\gamma\kappa(1+v')+\frac{1}{R_T}\right)\frac{\partial g}{\partial x}\right\} \quad (23)
\end{aligned}$$

By noting that $-\gamma\kappa(1+v')+\frac{1}{R_T} < 0$ and calculating $\frac{\partial g}{\partial x}, \frac{\partial h}{\partial x}$ for $x \in \{\eta, \Theta_T, \Theta_U\}$ gives the desired result. For example, the case for η gives,

$$\begin{aligned}
\frac{\partial g}{\partial \eta} &= \mathbb{E}(\tilde{S}_{c1} v_\eta)(1+v') + \mathbb{E}(\tilde{S}) v'_\eta \\
&= -\frac{\gamma v_\eta}{R_T} + \mathbb{E}(\tilde{S}) v'_\eta \\
\frac{\partial h}{\partial \eta} &= \mathbb{E}(\tilde{S}_{c1}(\tilde{R}-\mu)) v_\eta \\
&= \gamma \left(\frac{\mu}{R_T} - 1 \right) v_\eta > 0
\end{aligned}$$

resulting in,

$$\frac{\partial RP}{\partial \eta} = \frac{1}{c} \frac{\partial(\mu - R_T)}{\partial \eta} = \frac{-1}{|D|} \frac{1}{c} \left(\frac{\mu}{R_T} - 1 \right) \mathbb{E}(\tilde{S}) R_T \left\{ \gamma \kappa v_\eta v'' + v'_\eta \left(-\gamma\kappa(1+v') + \frac{1}{R_T} \right) \right\} < 0$$

The case for Θ_T stems from,

$$\begin{aligned}
\frac{\partial g}{\partial \Theta_T} &= \mathbb{E}(\tilde{S}_{c1})(1+v')^2 R_T - \mathbb{E}(\tilde{S}_{c0})(1+v') + v''\mathbb{E}(\tilde{S}) R_T \\
&= -\frac{\gamma}{R_T} \big((1+v') R_T + 1 \big) + v'' \mathbb{E}(\tilde{S}) R_T \\
\frac{\partial h}{\partial \Theta_T} &= \mathbb{E}(\tilde{S}_{c1}(\tilde{R}-\mu))(1+v') R_T - \mathbb{E}(\tilde{S}_{c0}(\tilde{R}-\mu)) \\
&= \gamma \left(\frac{\mu}{R_T} - 1 \right) \big((1+v') R_T + 1 \big)
\end{aligned}$$

where $\tilde{S}_{c0} = \gamma \tilde{S}$ since $u''(c) = -\gamma u'(c)$ resulting in,

$$\frac{\partial RP}{\partial \Theta_T} = \frac{1}{c} \frac{\partial(\mu - R_T)}{\partial \Theta_T} = \frac{-1}{|D|} \frac{1}{c} \left(\frac{\mu}{R_T} - 1 \right) \mathbb{E}(\tilde{S}) R_T v'' (1 + \gamma\kappa) < 0$$

And finally, for the case for Θ_U stems from,

$$\begin{aligned}
\frac{\partial g}{\partial \Theta_U} &= \mathbb{E}(\tilde{S}_{c1}(\tilde{R}-\mu))(1+v') + \mathbb{E}(\tilde{S}_{c1})(1+v')^2 \mu - \mathbb{E}(\tilde{S}_{c0})(1+v') + v'' \mathbb{E}(\tilde{S}) \mu \\
&= \gamma \left(\frac{\mu}{R_T} - 1 \right)(1+v') - \frac{\gamma}{R_T} \big((1+v')\mu + 1 \big) + v'' \mathbb{E}(\tilde{S}) \mu \\
\frac{\partial h}{\partial \Theta_U} &= \mathbb{E}(\tilde{S}_{c1}(\tilde{R}-\mu)^2) + \mathbb{E}(\tilde{S}_{c1}(\tilde{R}-\mu))(1+v')\mu - \mathbb{E}(\tilde{S}_{c0}(\tilde{R}-\mu)) \\
&= -\gamma \hat{V} + \gamma \left(\frac{\mu}{R_T} - 1 \right) \big((1+v')\mu + 1 \big)
\end{aligned}$$

resulting in,

$$
\begin{aligned}
\frac{\partial RP}{\partial \Theta_U} &= \frac{1}{c}\frac{\partial(\mu - R_T)}{\partial \Theta_U} \\
&= \frac{-1}{|D|}\frac{1}{c}\left\{-\gamma\left(-\gamma\kappa(1+v') + \frac{1}{R_T}\right)\left[\hat{V} - \left(\frac{\mu}{R_T} - 1\right)^2 (1+v')R_T\right]\right. \\
&\quad \left. +\mathbb{E}(\tilde{S})R_T v''\left(-\gamma\hat{V}\kappa + \left(\frac{\mu}{R_T} - 1\right)\left(\frac{\mu}{R_T} + \gamma\kappa\right)\right)\right\}
\end{aligned}
$$

■

Proof of Proposition 4:

Given that the equilibrium characterization of Proposition 1 is identical to that of Proposition 3, the partial derivatives of the new equilibrium equations are the same as before with a mere change in notation. Specifically, $R_T = R_{CB}$ and $\Theta_T = \Theta_{CB}$. Defining,

$$
\begin{aligned}
\overline{T}_1 &= \mathbb{E}(\tilde{S})(1 + v'(M;\eta))R_{CB} - 1 = 0 \\
\overline{T}_2 &= \mathbb{E}(\tilde{S}(\tilde{R} - \mu)) + \mathbb{E}(\tilde{S})(1 + v'(M;\eta))\mu - 1 = 0
\end{aligned}
$$

and labeling $g := \mathbb{E}(\tilde{S})(1 + v'(M;\eta))$ and $h := \mathbb{E}(\tilde{R}(\tilde{S} - \mu))$ the above observation implies that the expressions of $\frac{\partial g}{\partial x}, \frac{\partial h}{\partial x}$ as the same as for Proposition 1 for all variables x in the model. Therefore.,

$$
\begin{aligned}
|\overline{D}| &= \frac{\partial \overline{T}_1}{\partial \Theta_{CB}}\frac{\partial \overline{T}_2}{\partial \mu} - \frac{\partial \overline{T}_1}{\partial \mu}\frac{\partial \overline{T}_2}{\partial \Theta_{CB}} \\
&= \left(\frac{\partial g}{\partial \Theta_{CB}}\frac{\partial h}{\partial \mu} - \frac{\partial g}{\partial \mu}\frac{\partial h}{\partial \Theta_{CB}}\right)R_{CB} + \frac{\partial g}{\partial \Theta_{CB}} g R_{CB} \\
&= -\gamma^2\frac{R_{CB}}{c}((1+v') + \frac{1}{R_{CB}})\left(\hat{V} - \left(\frac{\mu}{R_{CB}} - 1\right)^2 (1+v')R_{CB}\right) - \gamma\left((1+v') + \frac{1}{R_{CB}}\right) \\
&\quad +\mathbb{E}(\tilde{S})v'' R_{CB}\left[\gamma\frac{\hat{V}}{c}R_{CB} - \gamma\left(\frac{\mu}{R_{CB}} - 1\right)\frac{\partial M}{\partial \mu} + 1\right]. \qquad (24)
\end{aligned}
$$

As before, the final equality stems from using $u''(c) = -\gamma u'(c)$ in order to express derivatives of the SDF and use the equilibrium equations $\overline{T}_1, \overline{T}_2$. The term accompanying v'' may have an ambiguous sign, but for v'' sufficiently small, it can be directly seen that $|\overline{D}|$ is negative. In this equilibrium we have $\frac{\partial \mu}{\partial \eta} = \frac{1}{c}\frac{\partial \mu}{\partial \eta}$, therefore we have,

$$\begin{aligned}\frac{\partial\mu}{\partial\eta} &= -\frac{c}{\overline{\overline{D}}}\left[\left(\frac{\partial g}{\partial\Theta_{CB}}\frac{\partial h}{\partial\eta} - \frac{\partial g}{\partial\eta}\frac{\partial h}{\partial\Theta_{CB}}\right)R_{CB}\right] \\ &= \gamma\left(\frac{\mu}{R_{CB}}-1\right)\mathbb{E}(\tilde{S})R_{CB}\left[v''v_\eta - v'_\eta\left((1+v')+\frac{1}{R_{CB}}\right)\right]\end{aligned}$$

which is clearly positive. Turning to changes in central bank repo take up gives,

$$\begin{aligned}\frac{\partial\Theta_{CB}}{\partial\eta} &= -\frac{c}{\overline{\overline{D}}}\left[\left(\frac{\partial g}{\partial\eta}\frac{\partial h}{\partial\mu} - \frac{\partial g}{\partial\mu}\frac{\partial h}{\partial\eta}\right)R_{CB} + \frac{\partial g}{\partial\eta}gR_{CB}\right] \\ &= \frac{-1}{|\overline{D}|}\left\{v_\eta\left(-\frac{\gamma^2}{c}\left[\hat{V}-\left(\frac{\mu}{R_{CB}}-1\right)^2(1+v')R_{CB}\right] - v''\gamma R_{CB}\mathbb{E}(\tilde{S})\left(\frac{\mu}{R_{CB}}-1\right)\frac{\partial M}{\partial\mu} - \frac{\gamma}{R_{CB}}\right)+\right. \\ &\quad \left.\mathbb{E}(\tilde{S})R_{CB}v'_\eta\left(\frac{\gamma\hat{V}}{c}+\gamma\left(\frac{\mu}{R_{CB}}-1\right)(1+v')\frac{\partial M}{\partial\mu}+\frac{1}{R_{CB}}\right)\right\}\end{aligned}$$

To sign the term accompanying v'_η note that $\frac{\partial M}{\partial\mu} = \Theta_U - w - 2(\mu - R_{CB})\frac{1}{c}$ and that households holdings of Treasuries is given by $\Theta_U - w - (\mu - R_{CB})\frac{1}{c}$, which is positive. By separating $2(\mu - R_{CB})$ so that half creates households allocations in Treasuries and the other half is grouped with $\hat{V}$ to create $\hat{V} - \left(\frac{\mu}{R_{CB}} - 1\right)^2(1+v')R_{CB}$ implies that the overall expression is positive.

■