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Preprint · August 2022
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Excess Mortality in Germany 2020-2022

Christof Kuhbandner and Matthias Reitzner

Abstract

The present study estimates the burden of COVID-19 on mortality. The state-of-the-art method of actuarial science is used to estimate the expected number of all-cause deaths in 2020 to 2022 if there had been no pandemic. Then, the number of observed all-cause deaths is compared with this expected number of all-cause deaths, yielding the excess mortality in Germany for the pandemic years 2020 to 2022.

The expected number of deaths is computed using the period life tables provided by the Federal Statistical Office of Germany and the longevity factors of the generation life table provided by the German Association of Actuaries. In addition, the expected number of deaths is computed for each month separately and compared to the observed number, yielding the monthly development of excess mortality. Finally, the increase in stillbirths in the years 2020 to 2022 is examined.

In 2020, the observed number of deaths was close to the expected number with respect to the empirical standard deviation, approximately 4,000 excess deaths occurred. By contrast, in 2021, the observed number of deaths was two empirical standard deviations above the expected number, and even more than four times the empirical standard deviation in 2022. The cumulated number of excess deaths in 2021, and 2022 is about 100,000 deaths. The high excess mortality in 2021, and 2022 was almost entirely due to an increase in deaths in the age groups between 15 and 79 and started to accumulate only from April 2021 onwards. A similar mortality pattern was observed for stillbirths with an increase of about 9.4% in the second quarter and 19.6% in the fourth quarter of the year 2021.

Something must have happened in April 2021 that led to a sudden and sustained increase in mortality in the age groups below 80 years, although no such effects on mortality had been observed during the COVID-19 pandemic so far.

1 Introduction

In the last two years, the burden of the COVID-19 pandemic on mortality has been intensively discussed. Basically, since COVID-19 is an infectious disease that is caused by a new virus, it is expected that many people have died because of the new virus who otherwise would not have died. In fact, this expectation represents one of the central justifications for the taking of countermeasures against the spread of the virus. Due to this reason, several previous studies have tried to estimate the extent of the mortality burden that has been brought about by the COVID-19 pandemic.

At first glance, it seems obvious to simply estimate the burden of the COVID-19 pandemic on mortality based on the number of officially reported COVID-19-related deaths. However, this has been proven to be difficult due to several reasons.

1.1 Reported COVID-19-Deaths: The Problem

A first difficulty is the problem that it is unclear whether a reported COVID-death died because of a SARS-CoV-2-infection or only with a SARS-CoV-2-infection. For instance, according to a published analysis of the German COVID-19 autopsy registry from March 2020 to the beginning of October 2021 [1], only 86% of the autopsied deaths with a COVID-19 diagnosis died from COVID-19. In particular, a closer look at the diagnostics used in this study suggests that this may be an overestimation. For instance, 87 of the 1,095 autopsied persons with the autopsy result of an "unspecific cause of deaths" were excluded although such persons seem not to have died from COVID-19. In addition, 10% of the deaths treated as "died from COVID-19" died actually due to bacterial or fungal super-infections or due to therapy-associated reasons and are thus not directly caused by COVID-19. These examples highlight the general problem that the answer to the question whether COVID-19 was the actual cause of death depends on the used definition of 'causality'.

A second difficulty is that even if a person died from COVID-19, this does not rule out the possibility that the person would have died as well even if there had been no COVID-19 pandemic. Many of the people that have died from COVID-19 were highly frail [2], and these people might have died from other causes of deaths if they had not died from COVID-19. For instance, it has been shown that rhinovirus infections have a high mortality risk for vulnerable elderly people as well [3]. Thus, even if there had been no SARS-CoV-2-infection waves, these individuals might instead have died in one of the rhinovirus-infection waves. Accordingly, even if there is a large number of deaths that were caused by a SARS-CoV-2-infection, this would not necessarily mean that all these deaths are additional deaths that would not have occurred if there had been no COVID-19 pandemic.

1.2 All-Cause Mortality: Estimating the Burden of the COVID-19 Pandemic

An obvious way to solve such problems when estimating the burden of the COVID-19 pandemic on mortality is to compare the number of observed all-cause deaths independently of the underlying causes of deaths with the number of all-cause deaths that would have been expected if there had been no pandemic. If there is a new virus that causes additional deaths beyond what is usually expected, the number of observed all-cause deaths should be larger than the number of usually expected deaths, and the higher the number of observed deaths is above the number of usually expected deaths, the higher is the burden of a pandemic on mortality. In particular, beyond the advantage that the above-mentioned problems with the number of the reported COVID-19-related deaths are avoided, another advantage is that additional indirect negative impacts of a pandemic on mortality are covered as well, such as a possible pandemic-induced strain of the health care system.

Due to these reasons, it is not surprising that several attempts have been made to estimate the increase in all-cause mortality during the COVID-19-pandemic [4, 5, 7, 8, 9, 10, 11]. Since the death of a person is a clear diagnostic fact, and since highly reliable data on mortality are available for several countries, at first glance, one may expect that the question of whether more people have died during the COVID-19-pandemic than is usually expected can be clearly answered.

However, the existing attempts show very large differences in the estimated increase in all-cause mortality during the COVID-19-pandemic. This can be illustrated for Germany where highly reliable data on the number of all-cause deaths even at the level of individual days are available. The estimated increase in all-cause mortality during the pandemic years 2020 and 2021 varies from 203,000 additional deaths [5] to only 29,716 additional deaths [7, 8], and for the pandemic year 2020, it has even been estimated that less all-cause deaths have been observed than usually expected [9].

How can this large variability in the estimated increase in all-cause mortality be explained? The number of observed all-cause deaths is a clearly defined number (although it seems that even in Germany it is difficult to determine this number precisely, see Section 3). But the estimation of the usually expected deaths is relatively complex and entails several choices of mathematical models and parameters, and which can lead to large differences in the estimated values (for a detailed discussion, see Sections 1.4 to 1.7 and Sections 3 and 4 below).

1.3 The Present Study

Against this background, the present article has the objective to provide a best-practice method, the state-of-the-art method of actuarial science, to estimate the expected all-cause mortality using the example of all-cause deaths in Germany in the years 2020 to 2022. The underlying standard model in actuarial mathematics was already used by Euler and Gauß, modern developments take into account mortality trends and longevity factors. Using this method, the increase in all-cause mortality in Germany for the pandemic years 2020 to 2022 is estimated.

In addition, an overview and an evaluation of the model and parameter choices that must be made is provided. This demonstrates that the amount of increase in all-cause mortality varies depending on the chosen model and parameters.

As described above, there are several studies that have attempted to estimate the increase in mortality in Germany in 2020, 2021, and 2022 based on different methods [5, 7, 8, 9, 11]. However, there are several unanswered questions:

(1) Only one study [5], which examined only the year 2020, took into account the historical trend in mortality rates. We use the mathematical model provided by the German Association of Actuaries. In particular, this includes longevity factors, which are well established in actuarial science.

(2) Although in most of the studies age-standardized estimations were made, age-dependent differences in mortality increase were not examined in detail. We use most recent life tables provided by the Federal Statistical Office of Germany to calculate age-dependent expectations.

(3) In none of the previous studies, it was examined how much the mortality estimates depend on the underlying data and vary with different approaches. Here we state the data uncertainty and calculate the model and parameter sensitivity by comparing the results achieved using different life tables and longevity factors.

(4) In all of the previous studies except a recent study [6] concerning Austria, only the estimated increase in all-cause deaths was reported, without examining whether the estimated increase exceeds the usual variation in mortality found across previous years. We estimate the yearly empirical standard deviation which could be used to obtain confidence intervals.

(5) The increase in mortality over the course of the year has so far only been investigated for 2020 in two studies [5, 7]. The years 2021, and 2022 have not yet been investigated in this respect. Furthermore, no study has yet determined the increase in mortality over the course of the year for different age groups.

(6) Comparing the results to possible influencing factors: In none of the previous studies, possible factors that might contribute to the observed course of the increase in mortality were explicitly examined on a monthly base during the pandemic years 2020 to 2022.

(7) Monthly increase in the number of stillbirths in the years 2020 to 2022 in Germany: In all previous studies, the increase in mortality has only been examined for the age groups 0 and above. Whether changes in mortality are also found at the level of stillbirths has not been investigated so far.

As will be shown, a proper analysis of the increase in all-cause mortality reveals several previously unknown dynamics that will require a reassessment of the mortality burden brought about by the COVID-19 pandemic.

1.4 Estimating the Increase in All-cause Mortality: Population-Size and Historical-Trend Effects

There are two main effects that have to be taken into account when estimating the increase in all-cause mortality: effects of changes in the size of the population and effects of historical trends in mortality rates. To illustrate these effects and the resulting potential pitfalls, Fig. 1 shows for the over 80 years old population in Germany the number of deaths (Fig. 1A), the population size (Fig. 1B), and the mortality rate (i.e., percentage of deceased persons; Fig. 1C) for the years 2016 to 2021.

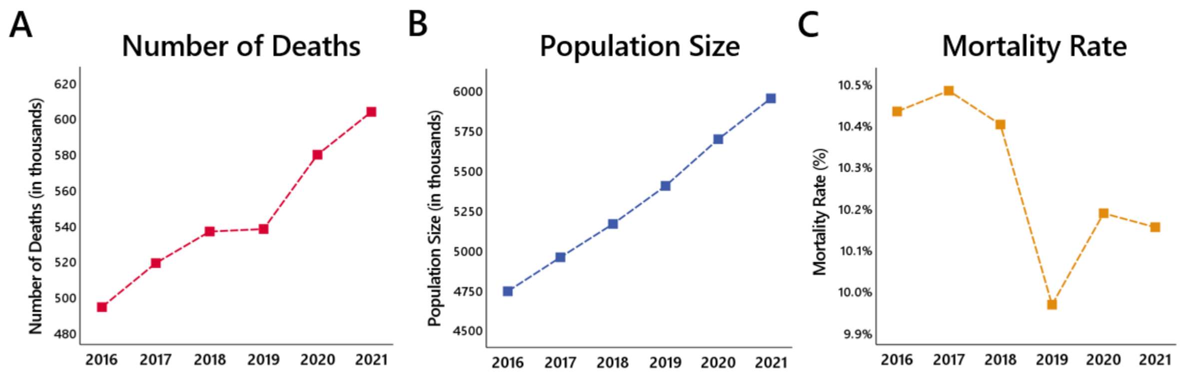

Fig. 1.: Population-size effects and historical-trend effects on the estimation of the increase in mortality. For the population over 80 years of age in Germany, (A) shows the number of deaths, (B) the population size, and (C) the mortality rate (i.e., percentage of deceased persons) for the years 2016 to 2021.

Changes in population size have to be taken into account due to the simple fact that the larger a population is, the more deaths occur. Ignoring existing changes in population size will lead to erroneous estimations. For instance, regarding the population over 80 years of age in Germany, the number of deaths increases from year to year (see Fig. 1A). Concluding from this pattern that mortality increased in the years 2020 and 2021 compared to previous years would make no sense because this increase is fully attributable to the increase of population size, as shown in Fig. 1B and 1C.

Historical trends in mortality rates have to be be taken into account due to the fact that mortality rates are not a stable values but influenced by environmental and societal changes and improvements in medical treatments. For instance, as can be seen exemplarily in Fig. 1C, in Germany, there is a historical trend of a continuous decrease in mortality rate that is observed in most age groups. If such a declining trend in mortality rates is not taken into account, the number of expected deaths is overestimated and thus the true mortality excess is underestimated.

The pitfall of ignoring changes in population size is for example found in the estimations provided by the German Federal Statistical Office [12] where the increase in mortality is estimated based on a comparison of the observed number of deaths with the median value of the four previous years. As illustrated in Fig. 2A, estimating the number of expected deaths based on the median of the four previous years underestimates the number of expected deaths and thus overestimates the true increase in mortality. The invalidity of this method can be illustrated by the fact that in case of a continuously increasing population size, as is the case for the population over 80 years of age in Germany, such a method would conclude for *every* year that there was an unexpected increase in mortality compared to previous years.

The pitfall of ignoring longer historic trends is for example found in the estimations provided by the World Health Organization (WHO) [11] where the increase in mortality is estimated based on a thin-plate spline extrapolation of the number of expected deaths. As illustrated in Fig. 2B, such an estimation method is highly sensitive to short-term changes in the observed number of deaths. Accordingly, erratic estimations of expected deaths predictions can occur. For instance, regarding the WHO estimations for

Germany, the spline extrapolation predicts – based on the short-term decline in deaths in 2019 compared to 2018 – that a similar decline would occur in the following years as well, although this completely contradicts the long-term historical trend.

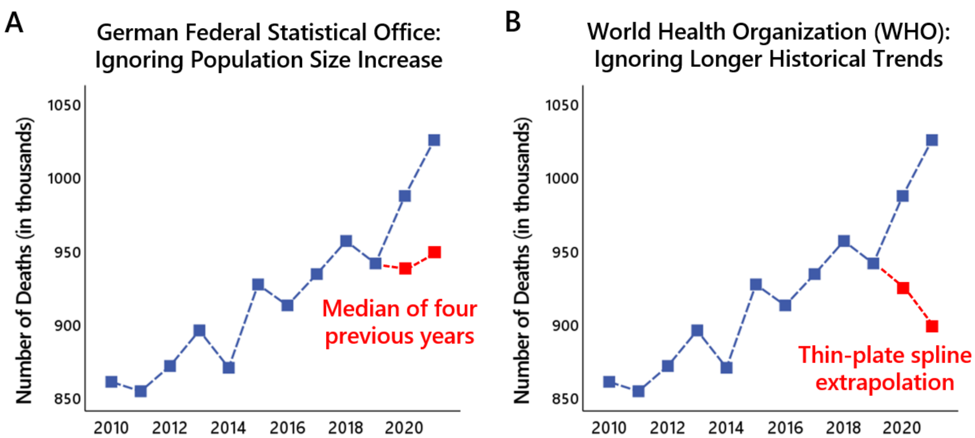

Fig. 2.: The pitfalls of ignoring population-size effects and historical-trend effects. The blue squares in (A) and (B) show the development of the number of deaths in Germany from 2010 to 2021 (all age groups). The red squares in (A) show the estimations of the number of expected deaths for the years 2020 and 2021 of the German Federal Statistical Office [12] which are based on the median of the four previous years. The red squares in (B) show the estimations of the number of expected deaths for the years 2020 and 2021 of the World Health Organization [11] which are based on a thin-plate spline extrapolation that is highly sensitive to short-time changes. As can be seen, both the ignoring of the increase in population size of the older age groups and the ignoring of longer historical trends leads to an underestimation of the expected deaths and thus to an overestimation of the true mortality increase in the years 2020 and 2021.

1.5 Methods That Take Into Account Population-Size and Historical-Trends Effects

A first and comparatively simple approach to take into account population-size and historical-trends effects is the attempt to predict the further course of the number of deaths from observed data in previous years using regression methods. For instance, in a study by Baum [4], the course of the observed increase in the number of deaths in Germany from 2001 to 2021 compared to the year 2000 was fitted with a polynomial function of order two, and the yearly residuals were used to estimate the yearly increase or decrease in mortality, resulting in an estimated increase in mortality in the years 2020 and 2021 of about 11,000 additional deaths each. While the advantage of this approach is on the one hand that no parameter choices have to be made as it is the case with the more complex estimation methods (see below), on the other hand this is at the same time the weakness of this approach: since every data point is given the same weight, unique outliers may lead to biased estimations, and developments depending on more complex circumstances cannot be incorporated in this approach.

To account for unique outliers, it has been tried to estimate the number of expected deaths by a time-series model based on the number of observed deaths in previous years, and to exclude past phases of unique excess mortality, as done in the EuroMOMO project [13]. However, beyond the problem that the resulting estimates depend on the specific model and parameter choices made (see below), a common problem for every approach that bases estimations on the raw number of observed deaths is that the resulting estimations do not take into account possible changes in the age structure within a population, which can lead to biased estimates.

To take into account changes in the age structure within a population, so-called age-adjustments has a long tradition in mortality research [14], which is essential especially when estimating the number of expected deaths in populations where the proportion of elderly people changes over time. The basic method is to compute mortality rates for a reference period separately for different age groups, and to extrapolate from the age-dependent mortality rates and the population sizes of the different age groups in the to-be-estimated year the number of expected deaths in each of the age groups.

An example is a recent study by Levitt, Zonta, and Ioannidis [10] where the increase in mortality in the years 2020 and 2021 was estimated based on the reference period of the three pre-pandemic years 2017-2019 using age strata of 0-14, 15-64, 65-75, 75-85, and 85+ years, resulting in an estimated increase in mortality of about 16,000 additional deaths in the year 2020, and 38,800 additional deaths in the year 2021. In two studies by De Nicola et al. [5,6], a more refined method (see below) and a more fine-grained age adjustment was used, resulting in even lower estimates of increased mortality with about 6,300 additional deaths in 2020 and 23,400 additional deaths in 2021.

A problem in both the study by Levitt et al [10]. and the studies by De Nicola et al. [7, 8] is that possible historical trends in mortality rates are not taken into account. This was, in addition to an age-adjustment, done in a study by Kowall et al. [9] where the increase in mortality in the year 2020 was estimated for the countries Germany, Spain, and Sweden. Historical trends in mortality rates were estimated based on the observed decrease in mortality rates in the pre-pandemic years 2016-1019. For Germany, it was estimated that the number of observed deaths in 2020 was 0.9% higher than the number of estimated expected deaths, which is in the range of the estimations in the De Nicola et. study. Estimations with adjustments for changes in historical trends in mortality rates for the year 2021 have to date not been reported, at least to our knowledge.

1.6 The Inherent Model Uncertainty of Estimates of Increases in Mortality

As has already become apparent in the previous paragraphs, the estimation of the amount of increase in all-cause mortality entails several model and parameter choices that have to be made. While a proper analysis necessarily requires the taking into account of changes in population sizes and historical trends in mortality rates, there remain a number of degrees of freedom how to exactly do this. For instance, an open question is which previous years are used as a reference and which model is used for the extrapolation of the expected deaths based on these years.

What a large effect a small change in the chosen perspective can have on the estimation of the amount of mortality increase is illustrated in Fig. 3 using the German mortality figures. When trying to estimate the increase in the number of all-cause deaths in the years 2020 and 2021 by a comparison with the number of expected deaths that is estimated based on the course of deaths in the four pre-pandemic years 2016-2019, one can for instance take two different perspectives: one can consider the year 2018 as an unusual outlier above the typical course of the number of deaths, or one can consider the year 2019 as an unusual outlier below the typical course of the number of deaths. Depending on the chosen perspective, extrapolating the expected number of deaths with either excluding the year 2018 ("outlier upwards") or the year 2019 ("outlier downwards") leads to totally different results, with an estimation of a strong increase in mortality in the former case and an estimation of even a slight decrease in mortality in the latter case.

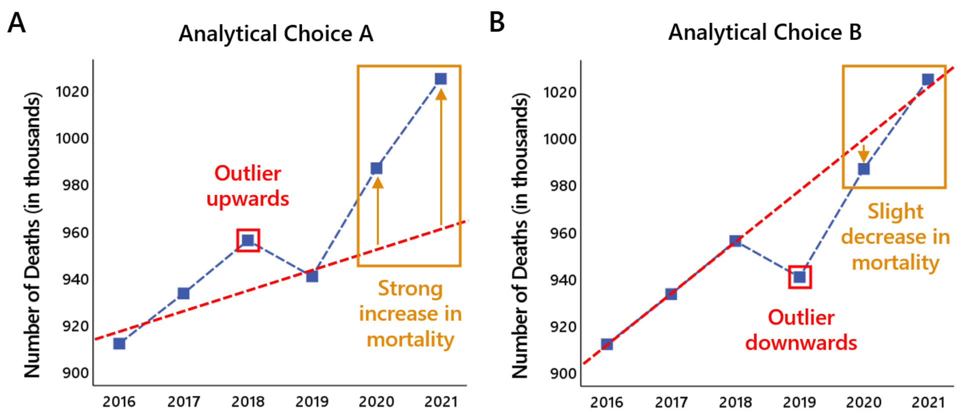

Fig. 3.: Possible large effect of small changes in the chosen perspective. The blue squares in (A) and (B) show the development of the number of observed all-cause deaths in Germany from 2016 to 2021 (all age groups). The expected all-cause deaths in the years 2020 and 2021 are estimated based on the observed deaths in the years 2016-2019 using a simple linear regression function, excluding the year 2018 as an "unusual outlier upwards" (A) or the year 2019 as an "unusual outlier downwards" (B), giving the impression of a strong increase in mortality in the former case and the impression of even a slight decrease in mortality in the latter case.

Since there is no truth criterion that would determine which of the choices is the best one to be made, there is no such thing as a "true" increase in mortality. Instead, the amount of increase in mortality must be understood as a construct with an inherent model uncertainty, that varies depending on the chosen perspective. This fact has at least three important implications:

First, when reporting estimates of the amount of increase in mortality, it is important to show how strongly the estimates vary with different model and parameter choices that can reasonably be made. In particular, possible choices and the resulting estimates should be communicated to readers in a way so that they are enabled to draw their own conclusions depending on their specific questions they would like to answer (see next point).

Second, when interpreting estimates of the increase in mortality, one has to be aware of the made model and parameter choices. In particular, when deciding which approach is chosen, one has to clarify which question is tried to answer, and to choose the approach that best fits the to-be-answered question. For instance, if one is interested in the question of how far the observed number of deaths is above the usually occurring deaths, excluding outlier years when estimating the amount of increase in mortality may be a reasonable decision. However, if one is interested in whether the observed number of deaths is above the extreme values of previous years, excluding outliers may be a less reasonable decision.

Third, despite the inherent uncertainty of the estimates of increases in mortality, the comparison of increases in mortality between different years may nevertheless reveal clear results. If the observed difference between the years does not vary as a function of the chosen parameters and model, it can be assumed that the observed differences in estimated increases in mortality reflects the true fact that there was a larger increase in mortality in one of the years.

1.7 The Use of the Term "Excess Mortality"

In many of the previous studies, the observation that the number of observed all-cause deaths is larger than the number of expected all-cause deaths is designated by the term "excess mortality". However, such a use of terms is questionable. The number of deaths from year to year does not follow a straight line but varies around a common trend.

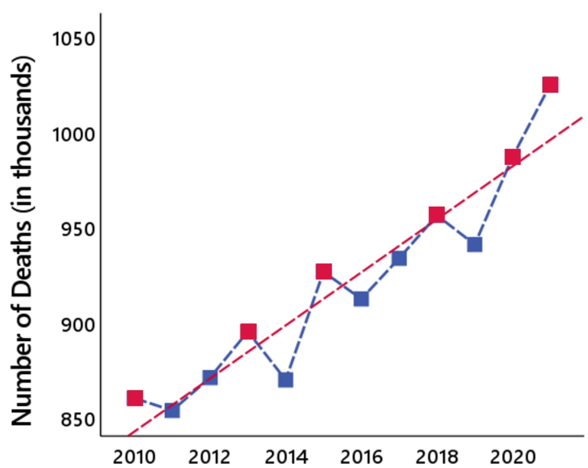

Fig. 4.: The inflationary use of the term "excess mortality". The colored squares show the number of all-cause deaths in Germany from 2010 to 2021. The dashed red line shows the common trend across the years (linear regression). If one were to designate as "excess mortality year" all years in which more deaths are observed than expected according to the common trend (red-colored squares), one would have to conclude that an "excess mortality" is observed in six years, and a "mortality deficit" in the other six years.

Accordingly, as illustrated in Fig. 4, if one were to designate as "excess mortality year" all years in which more deaths are observed than expected according to the common trend, one would have to conclude that an “excess mortality” is observed in about 50% of all years, and a “mortality deficit” in the other 50% of all years.

Since about half of the years show mortality levels above the common trend, one could use the term “excess mortality” only for years that show an outstanding increase in mortality above a certain threshold. One straightforward possibility to establish such a threshold would be to compute the mean variation (empirical standard deviation) around the common trend across the years, and to designate as “excess mortality years” only those in which the number of observed deaths exceeds twice the mean variation.

Another possibility would be to search for previous years with peak deviations from the common trend, and then to compare the deviation observed in the year one is interested in with the peak deviations in previous years. Such a comparison was for instance made in a recent study by Staub et al. [15] where the historical dimension of the COVID-19 pandemic was examined for the countries Switzerland, Sweden, and Spain over a time span of more than 100 years, revealing that the peaks of monthly excess mortality in 2020 were greater than most peaks since 1918.

Nevertheless, also in this contribution we decided to use the terms “excess mortality” and “mortality deficit” for a mortality which is just above, respectively below, the estimated value, as in most other contributions. An attempt to define an outstanding “excess mortality year“ via mean variations will be made in Section 3 and Section 4.

2 Yearly expected mortality

2.1 Methods

It is the standard method in actuarial science to use life tables and population tables, and to obtain by a (suitable modified) multiplication the expected number of deaths. Historical population tables are used to estimate the longevity trend which is in addition taken into account.

Thus, the starting point for our investigations are the period life tables and population demographics available from the Federal Statistical Office of Germany. As usual in actuarial science, we denote by

- $l_{x,t}$ the number of x year old male at January 1st in year t;
- $l_{y,t}$ the number of y year old female at January 1st in year t;
- $d_{x,t}$ the number of deaths of x year old males in year t;
- $d_{y,t}$ the number of deaths of y year old females in year t;
- $q_{x,t}$ (an estimate for) the mortality probability for an x year old male in year t;
- $q_{y,t}$ (an estimate for) the mortality probability for a y year old female in year t.

Note that $d_{x,t}$ also contains deceased that have been $(x-1)$ years old at January 1st in year t and died as x year old. To compensate this problem, the 2017/2019 life table of the Federal Statistical Office of

Germany [16] (like most German life tables) uses the method of Farr to estimate $q_{x,t}$ (and analogously $q_{y,t}$).

$$\hat{q}_{x,2019} = \frac{\sum_{t=2017}^{2019} d_{x,t}}{\frac{1}{2}\sum_{t=2017}^{2019}(l_{x,t} + l_{x,t+1}) + \frac{1}{2}\sum_{t=2017}^{2019} d_{x,t}} \tag{1}$$

The period life table 2017/19 of the Federal Statistical Office of Germany thus takes into account only the mortality probabilities in these three years.

A much more complicated task is to compute generation life tables. Generation life tables observe the mortality development over a long period, roughly 100 years, smoothen the existing data, and in particular estimate the long term behaviour of the mortality probabilities. These probabilities have been decreasing within the last 100 years, and the common ansatz is to set

$$q_{x,t} = q_{x,t_0} e^{-F(x;t,t_0)}, \; q_{y,t} = q_{y,t_0} e^{-F(y;t,t_0)}.$$

Here, the German Association of Actuaries (DAV) uses a smoothed life table q_{x,t_0} in the base year t_0, and models the trend underlying future mortality, the longevity trend function $F(x;t,t_0)$, via regression separately for the male and female population. In the year 2004, it turned out that the decrease of the mortality probabilities in the previous years has been steeper than expected, therefore the DAV life table DAV 2004 R [17] distinguishes between a higher short-term trend and a lower long-term trend. These trends are of high importance and used for life annuities, whereas for life insurances the trend (at least the short term trend) is mostly ignored.

One should keep in mind that the life tables DAV2004R and the longevity factors DAV2004R are tailor made for pensions funds. Since we are interested in predictions concerning the whole German population, we use the life table for the general population of the Federal Statistical Office of Germany, not the life table DAV2004R, and adapt the longevity factors of the DAV2004R to fit for the whole population. In addition, it seems that the longevity trend was flattening in the last years. Therefore, we have decided to use half the long-term trend function given by the DAV2004R,

$$F(x;t,t_0) = \frac{1}{2}(t - 2019)F_{l,x}, \; F(y;t,t_0) = \frac{1}{2}(t - 2019)F_{l,y}$$

where the numbers $F_{l,x}$ and $F_{l,y}$ are contained in the DAV 2004 R table. We also decided to use the probabilities $\hat{q}_{x,2019}$ and $\hat{q}_{y,2019}$ of the life table 2017/2019 by the Federal Statistical Office of Germany as the base life table in a first step, and thus take $t_0 = 2019$. A second possible choice would be to take $t_0 = 2018$, the 'mean year' of the table, which only results in minor changes, but we follow the actuarial standard to set t_0 as the year when the table was completed.

For a discussion concerning our model parameters, i.e. the influence of the longevity trend and our choice using half of it, and the choice of the (non-smoothed) life table 2017/19, we refer to Section 3. Also, it is well known that mortality probabilities for males and females differ substantially, therefore these two cases are computed separately.

Putting things together, we define the mortality probability of an x year old male in year t by

$$q_{x,t} = \hat{q}_{x,2019} e^{-\frac{1}{2}(t-2019)F_{l,x}},$$

and for a y year old female in year t by

$$q_{y,t} = \hat{q}_{y,2019} e^{-\frac{1}{2}(t-2019)F_{l,y}}.$$

Now, for each individual, the probability to die at age x is given by $q_{x,t}$, and hence, in a first attempt, a population of $l_{x,t}$ individuals produces binomial distributed random numbers $D_{x,t}$ and $D_{y,t}$ of deaths for males, respectively females, with expected values

$$\mathbb{E}D_{x,t} = l_{x,t} q_{x,t}, \; \mathbb{E}D_{y,t} = l_{y,t} q_{y,t}.$$

As is well known (and already discussed above in connection with Farr's method), this formula ignores those individuals which have been of age $(x-1)$ at the beginning of year t, and died as x year olds. To compensate for this missing piece, we follow the procedure proposed by De Nicola et al. [7]. Roughly half of the $x-1$ year old population at the beginning of the year which is of size $l_{x-1,t}$ dies after its birthday as x year old. For them we use the smoothed mortality probability

$$\frac{q_{x-1,t} + q_{x,t}}{2}.$$

The other half of the x year old deceased belongs to the population of x year old at the beginning of the year which is of size $l_{x,t}$. For them we use the smoothed mortality probability

$$\frac{q_{x,t} + q_{x+1,t}}{2}.$$

For more details see [7]. Hence, for $x = 0, \ldots, 101$ the random number $D_{x,t}$ of deaths of age x in year t is binomial distributed and satisfies

$$\mathbb{E}D_{x,t} = \frac{1}{2}\left(l_{x-1,t} \frac{q_{x-1,t} + q_{x,t}}{2} + l_{x,t} \frac{q_{x,t} + q_{x+1,t}}{2} \right) \tag{2}$$

where $l_{x-1,t}$ and $l_{x,t}$ are taken from the population table of the Federal Statistical Office of Germany [18]. For $x = 0$ we set $l_{-1,t} = l_{0,t+1}$ if available, $l_{-1,t} = l_{0,t}$ else, and $q_{-1,t} = q_{0,t}$. The same considerations lead to $\mathbb{E}D_{y,t}$.

The 2017/2019 life table by the Federal Statistical Office of Germany contains the mortality probabilities $q_{x,t}$ and $q_{y,t}$, and the underlying population table the population size $l_{x,t}$ and $l_{y,t}$ for the age $x = 0, \ldots, 100$. In principle it would be more precise to use life tables and population tables up to age 113 but these data are not available. The excess mortality is obtained by comparing the expected values $\mathbb{E}D_{x,t} + \mathbb{E}D_{y,t}$ to the observed data $d_{x,t} + d_{y,t}$ for $t = 2020, 2021$ and 2022 .

Some remarks are important in order to contextualize the method.

- Modelling the longevity factors is a challenging task. For example, the Actuarial Association of Austria uses factors involving arctan $\left(\frac{t}{100} - 20.01\right)$ which has serious advantages. The need for longevity factors depends heavily on the country, it seems for example that in Japan and in England the mortality trend has already vanished and the mortality probabilities are more or less constant.

- The mortality probability heavily depends on gender and differs for the male and female population. However, the resulting excess mortality is nearly the same for the male and female population. Hence, in the following, we calculate the expected number of deaths separately and show only the total number of deaths. On the other hand, huge differences occur for the excess mortality in different age groups, and therefore we present our results for each age group separately.

- The mortality probability not only depends on age and gender, but also significantly on social status, profession, health condition, region, etc. As is common, the German life tables give average mortality probabilities. Also, it is unclear - at least to the authors - whether the SARS-CoV-2-infection rate and mortality depends on these factors, too. For a deeper investigation of COVID-19 mortality increase, this should be taken into account, but at the moment, appropriate data are not available.

- One has to take into account that the year 2020 is a leap year. Therefore we have "added" an additional day by multiplying the result of the computations described above by $\frac{366}{365}$.

2.2 Results

Following the computations described in the previous section, we obtain the expected number of deaths in 2020, 2021, and 2022. The expectations $\mathbb{E}D_{x,t}$ and $\mathbb{E}D_{y,t}$ for each age $x, y = 0, 1, \ldots, 99$ and $t = 2020$, 2021, and 2022 are given in the supplement, Section 8.1. The Federal Statistical Office of Germany provides the number of deaths only in certain age groups [19], and for the year 2022 the number of deaths reported by the Federal Statistical Office of Germany is still preliminary. The following table gives the number of deaths in the age groups

$$\bar{a} \in \{[0,14], [15,29], [30,39], [40,49], [50,59], [60,69], [70,79], [80,89], [90,\infty)\}.$$

We set

$$D_{\bar{a},t} = \sum_{x \in \bar{a}} D_{x,t} + \sum_{y \in \bar{a}} D_{y,t} \quad \text{and} \quad d_{\bar{a},t} = \sum_{x \in \bar{a}} d_{x,t} + \sum_{y \in \bar{a}} d_{y,t}.$$

To compare the expected $\mathbb{E}D_{\bar{a},t}$ and the observed values $d_{\bar{a},t}$, we use the relative difference

$$\frac{d_{\bar{a},t} - \mathbb{E}D_{\bar{a},t}}{\mathbb{E}D_{\bar{a},t}}.$$

Table 1: Expected deaths and yearly excess mortality for different age groups.

	$t = 2020$			$t = 2021$			$t = 2022$		
age range	expected observed	abs. diff.	rel. diff.	expected observed	abs. diff.	rel. diff.	expected observed	abs. diff.	rel. diff.
0-14	3,531			3,513			3,517		
	3,306	-225	-6.38%	3,368	-145	-4.14%	3,527	10	0.28%
15-29	3,944			3,817			3,755		
	3,844	-100	-2.53%	3,934	117	3.07%	4,115	360	9.59%
30-39	6,626			6,585			6,546		
	6,668	42	0.64%	6,812	227	3.44%	7,130	584	8.92%
40-49	15,345			14,877			14,601		
	15,507	162	1.06%	16,095	1,218	8.19%	15,653	1,052	7.20%
50-59	58,641			57,705			56,471		
	57,331	-1,310	-2.23%	59,350	1,645	2.85%	56,554	83	0.15%
60-69	117,432			118,456			119,983		
	118,460	1,028	0.88%	126,781	8,325	7.03%	128,370	8,387	6.99%
70-79	198,389			190,335			186,303		
	201,957	3,568	1.80%	204,839	14,504	7.62%	205,435	19,132	10.27%
80-89	378,459			392,535			404,994		
	378,406	-53	-0.01%	398,041	5,506	1.40%	421,201	16,207	4.00%
90-∞	199,191			201,884			202,375		
	200,093	902	0.45%	204,467	2,583	1.28%	219,191	16,816	8.31%
total	981,557			989,707			998,545		
	985,572	4,015	0.41%	1,023,687	33,980	3.43%	1,061,176	62,631	6.27%

The deviation in 2020, 2021, and 2022 must be compared to the deviation inherent in the parameter choice of our model, and the empirical standard deviation which has occurred in the years before. This will be done in Section 3 and Section 4 for the total number and for all age groups separately. It will turn out, that in year 2020 the observed numbers of deaths are extremely close to the expected numbers with respect to the empirical standard deviation. Yet, in 2021 the difference between the observed total number of deaths and the expected number is more than twice the empirical standard deviation, and in 2022 it is even beyond four times the standard deviation. This mortality excess is mainly caused by the following age groups: in 2021 we observe an excess mortality of more than twice the empirical standard deviation in the age groups $[60, 69]$, $[70, 79]$, and an excess mortality of five times the empirical standard deviation in the age group $[40, 49]$; in 2022 we observe an excess mortality of more than twice the empirical standard deviation in the age groups $[15, 29]$, $[30, 39]$, $[60, 69]$, $[70, 79]$, $[80, 89]$, and an excess mortality of four times the empirical standard deviation in the age groups $[40, 49]$, $[90, \infty)$. The other age groups are below twice the standard deviation.

Fig. 5 illustrates that the deviation of the observed mortality from the expected mortality is not uniform over the different age groups, and, in particular, that the structure changes from 2020 to 2021, and 2022. A closer look reveals that the excess mortality observed in 2021 is almost entirely due to an above-average increase in deaths in the age groups between 15 and 79. The highest values are reached in the age group 40-49, where an increase in the number of deaths is observed that is 9% higher than the expected values. In 2022 the excess mortality is above 6% for nearly all age groups in the range $[15, \infty)$. An exception for all three years is the age group $[50, 59]$ where, in contrast to the surrounding age groups, a substantially lower excess mortality is observed. This is also visible if the 2017/2019 life table by the Federal Statistical Office of Germany is replaced by a life table from another year. We are not aware of an explanation for this fact, an interesting avenue for future research may be to explore what factors make this age group so resilient.

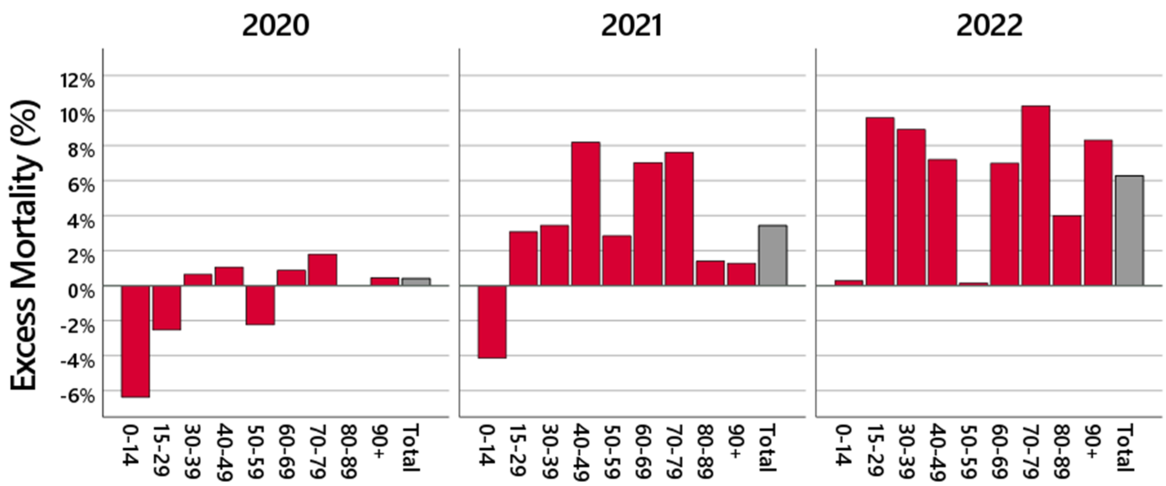

Fig. 5: Yearly excess mortality [1]. The red bars show the excess mortality in 2020 (left panel), 2021 (middle panel), and 2022 (right panel) in different age groups, the grey bars the total excess mortality.

It should be pointed out that in the last twenty years the maximal excess mortality in a year was about 25,000 deaths, and the authors are not aware of an excess mortality of more than 60,000 deaths – or in two consecutive years about 100,000 deaths – in the last decades.

[1] For infants something unexplained happens. In the beginning of 2020 there were 774,870 people of age 0, during the year 2,373 children of age 0 died, yet at the end of 2020 there were 783,593 (!) people of age 1. This is maybe due to migration effects, but we do not have sufficient precise data to model this effect. And for our investigations concerning COVID-19 excess mortality, the infant mortality can be ignored.

3 Data uncertainty and model uncertainty

The most basic data set for estimating excess mortality is the number of all-cause deaths in each year. The Federal Statistical Office of Germany publishes each week the number of reported deaths. After the end of the year, the Federal Statistical Office of Germany undertakes a "plausibility check" and then publishes about September next year the corrected final number of deaths. E.g., for 2019, this resulted in a change of at least 20,000 data sets yielding a cumulative change of nearly 3,000 deaths, and for 2021 we observe a cumulative change of more than 2,000 deaths. Hence, even in a country like Germany, already the number of observed deaths seems to have an intrinsic uncertainty of 2,000 to 3,000 deaths. One should keep in mind that the life tables of the Federal Statistical Office of Germany are calculated using this reported number of deaths, and hence also the life tables contain this data uncertainty.

Computing the expected number of deaths using a life table, several parameters for modelling mortality probabilities essentially influence the results. One could replace the 2017/2019 life table of the Federal Statistical Office of Germany by the life tables 2016/18 or 2015/17. And one could use different longevity factors, or ignore them totally. The question, whether a serious excess mortality occurs for 2020, 2021, and 2022, heavily depends on this underlying data sets. In the next table we present the total expected number of deaths over all age groups

$$\mathbb{E}D_t = \sum_{x=0}^{101} \mathbb{E}D_{x,t} + \sum_{y=0}^{101} \mathbb{E}D_{y,t}$$

using different life tables and taking into account either none, or half, or the full longevity trend.

Table 2: Expected deaths for different life tables.

longevity trend	life table	$\mathbb{E}D_{2020}$	$\mathbb{E}D_{2021}$	$\mathbb{E}D_{2022}$
none	2015/17	1,010,478	1,025,768	1,041,319
	2016/18	999,592	1,014,802	1,030,423
	2017/19	988,288	1,003,270	1,018,827
half	2015/17	989,964	998,213	1,006,620
	2016/18	986,021	994,294	1,002,869
	2017/19	981,557	989,707	998,545
full	2015/17	969,896	971,451	973,159
	2016/18	972,649	974,230	976,105
	2017/19	974,875	976,341	978,263
	observed	985,572	1,023,687	1,061,176

It turns out that the life tables have a significant effect on the question whether an excess mortality exists. For example, the use of the life table 2015/17 of the Federal Statistical Office of Germany without the longevity trend yields for the first two Corona-years 2020 and 2021 even a mortality deficit. And when keeping half the longevity trend, in 2021 the excess mortality of 31,723 deaths for the life table 2017/19 should be compared to the smaller excess mortality of 23,217 deaths when using the life table 2015/17, the total difference being 8,506 deaths. In other words, the life tables of the Federal Statistical Office of Germany have a serious fluctuation over the years which should be taken into account as the model uncertainty. Yet all parameter choices lead to a serious mortality excess for the year 2022.

For a more convenient view we present the excess mortality using the relative difference, see Table 3 and Fig. 6.

Table 3: Excess mortality for different life tables.

longevity trend	life table	2020	2021	2022
none	2015/17	-2.46%	-0.20%	1.91%
	2016/18	-1.40%	0.88%	2.98%
	2017/19	-0.27%	2.04%	4.16%
half	2015/17	-0.44%	2.55%	5.42%
	2016/18	-0.05%	2.96%	5.81%
	2017/19	0.41%	3.43%	6.27%
full	2015/17	1.62%	5.38%	9.04%
	2016/18	1.33%	5.08%	8.72%
	2017/19	1.10%	4.85%	8.48%

In the light of these results, we have decided to choose a model which avoids the extremes and includes half of the longevity factor in Section 2.1. In this case, the range between the three models – which is an indicator for the model uncertainty – is in both years approximately 8,500 deaths per year.

Yet in all these results obtained by life tables of recent years of the Federal Statistical Office of Germany, and in most other models [4, 5, 7, 8, 9], the main point coincides with our results: for 2020 the number of deaths is close to the expected value, whereas for 2021 there is a noticeable excess mortality, and for 2022 there is a huge mortality excess which has not been observed during the last decades.

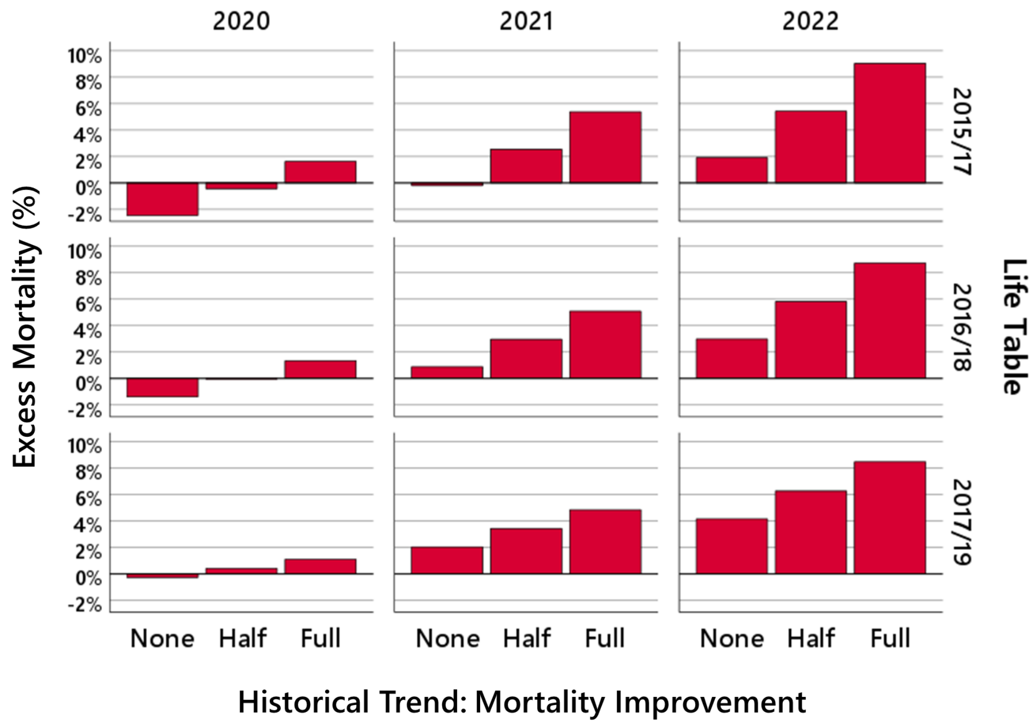

Fig. 6: The model sensitivity. The bars show the mortality deficit, respectively the excess mortality in 2020 (left panel), 2021 (middle panel), and 2022 (right panel) for different life tables and longevity trends.

A more detailed analysis of all the age groups introduced in Section 2.2 shows that, independently of the model used, the excess mortality in 2021, and 2022 is far above the values for 2020. These more detailed results are given in the supplement, Section 8.2.

4 The empirical standard deviation

As remarked in Section 2.2, to contextualize the deviation in 2020 – 2022, it must be compared to the model uncertainty, and to the empirical standard deviation occurred in the years before. Since the precise value of the empirical standard deviation – like the expectation – heavily depends on the underlying mathematical model, and since we are only interested in a rough approximation of the empirical standard deviation, we use an extremely simple model: we approximate the expected number of deaths using a linear regression model and calculate the empirical standard deviation in this model.

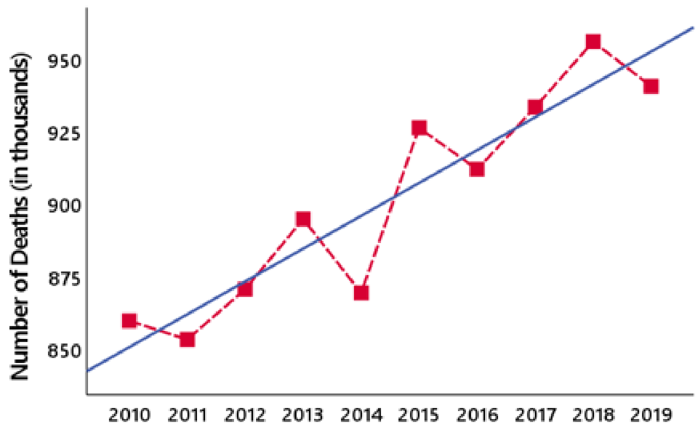

Fig. 7: The empirical standard deviation. The red squares show the number of all-cause deaths in Germany from 2010 to 2019. The blue line shows the regression line.

The regression leads to

$$d_t = \sum_{x=0}^{100} d_{x,t} + \sum_{x=0}^{100} d_{y,t} \approx L(t) = -21,936,713.9 + 11,336.2 \cdot t$$

which shows that each year we expect an increase of approximately 11,300 deaths in Germany. Observe that we have taken into account that the years 2012 and 2016 have been leap years and the number of deaths has been normalized to 365 days per year.

Table 4: Linear regression of the observed deaths.

year t	lin. reg. $L(t)$	observed d_t
2010	849,062	858,768
2011	860,398	852,328
2012	871,735	867,206
2013	883,071	893,825
2014	894,407	868,356
2015	905,743	925,200
2016	917,079	908,410
2017	928,416	932,263
2018	939,752	954,874
2019	951,088	939,520

Calculating in this simple model the empirical standard deviation gives

$$\hat{\sigma} = \hat{\sigma}(d_t) = 14{,}162. \tag{3}$$

We do not claim that this is a precise estimate of the standard deviation $\sigma(D_t)$, yet we are convinced that this at least reflects the order of magnitude. To check whether this order of magnitude is plausible, we also computed the empirical standard deviation for the years 2000-2009, using again the linear regression model. For these years, the empirical standard deviation is approximately $12,600$ which is the same order as (3).

At first sight this empirical standard deviation is somehow surprising and seems to be in contrast to the model used for modelling $D_{x,t}$ described in Section 2.1. As is common, we assumed that the number of deaths follows a binomial distribution. This is the most natural assumption. It would imply that the variance $\mathbb{V}D_{x,t} = l_{x,t}(1 - q_{x,t})q_{x,t}$ is approximately the number of deaths $l_{x,t}q_{x,t}$, since for the large majority of x the mortality probabilities are close to zero. This assumption and the independence property of the binomial model would lead to a total variance of approximately one million, and a standard deviation of approximately 1,000 in Germany. Thus, in actuarial science, a further randomization of $q_{x,t}$ is introduced which keeps the expectation unchanged – and thus our results in Sections 2.1–3 are still valid – but increases the variance to the observed 14,000.

We compare the excess mortality of approximately 4,000 deaths in 2020, 34,000 deaths in 2021, and 63,000 in 2022 to the empirical standard deviation $\hat{\sigma}$. In 2020, this leads to

$$d_{2020} - \mathbb{E}D_{2020} \approx 0.28\hat{\sigma},$$

hence the number of deaths in 2020 is very close to the expected number. For 2021, we have

$$d_{2021} - \mathbb{E}D_{2021} \approx 2.40\hat{\sigma}$$

and for 2022

$$d_{2022} - \mathbb{E}D_{2022} \approx 4.42\hat{\sigma}.$$

In many applications, an observed deviation beyond twice the standard deviation is called 'significant' because for normal distributed random variables the 5% confidence interval leads to this bound. For a normal distributed random variable, a bound of 2.4 times the standard deviation (occurring in 2021) leads approximately to a 1.6% confidence interval, and a bound of 4.4 times the standard deviation (occurring in 2022) leads approximately to a 0.006% confidence interval. We want to point out that the probability distribution of the number of deaths as a random variable is unknown and thus we avoid the use of the words "confidence interval" for excess mortality. Comparing the excess deaths against the empirical standard deviation just enables the reader to compare the deviation in 2020, 2021, and 2022 to historical fluctuations.

In addition one should also have in mind the data uncertainty of 2,000 to 3,000 deaths, and the model uncertainty of approximately 4,250 deaths.

The same method can be applied to certain age groups $\bar{a}$, using a linear regression model only for the development of the number of deaths in this age group, and estimating the observed empirical variance $\hat{\sigma}(d_{\bar{a},t})$. This leads to the following results, for the details see Section 8.3 in the supplement.

Table 5: Empirical standard deviations for age groups $\bar{a}$.

age range	$\hat{\sigma}(d_{a,t})$
0-14	158
15-29	148
30-39	245
40-49	237
50-59	868
60-69	3,646
70-79	6,101
80-89	7,770
90-∞	4,005
total	14,162

Comparing these to the values in Table 1 shows that the excess mortality in 2021 is more than twice the empirical standard deviation in the age groups $[40, 49], [60, 69], [70, 79]$, and in 2022 more than twice the empirical standard deviation in *all* age groups except $[0, 14]$ and $[50, 59]$, whereas in 2020 for all age groups the excess deaths are close to the expected value compared to the empirical standard deviation.

5 Monthly expected mortality

5.1 Methods

In the following two sections, we present a more detailed analysis of the number of deaths during the years 2020 to 2022. It is well known that the mortality probabilities are not constant but differ from month to month with peaks at the beginning and the end of the year, and also sometimes in summer when the weather is too hot (and depending on many other circumstances).

Unfortunately, the data basis for such investigations provided by the Federal Statistical Office of Germany is somehow weak. Therefore, again several approximation steps have to be applied. We denote by $d_{x,t,m}$, respectively $d_{y,t,m}$, the number of deaths of x year old male and y year old female in year t in month m. The Federal Statistical Office of Germany offers tables for $d_{\bar{x},t,m}$ and $d_{\bar{y},t,m}$ in the age groups $\bar{x}, \bar{y} \in \{[0, 14], [15, 29], [30, 34], [35, 39], \ldots, [90, 94], [95, \infty)\}$ which we use for the years $t = 2010, \ldots, 2022$, see [19].

Denote by f_m the estimated proportion of deaths in month m, $m = 1, \ldots, 12$. I.e., we distribute $d_{\bar{x},t}$

onto the monthly number of deaths $d_{\bar{x},t,m}$ via

$$f_{\bar{x},m} = \frac{1}{10}\sum_{t=2010}^{2019}\frac{d_{\bar{x},t,m}}{d_{\bar{x},t}}, \quad \sum_{m=1}^{12} f_{\bar{x},m} = 1,$$

where we modify the formula slightly to take into account that 2012 and 2016 were leap years. We list the obtained estimates in the supplement, Section 8.4. These mortality factors have been highly concentrated around their mean $f_{\bar{x},m}$ during the last years, the empirical standard deviation being below 1.5% for all age groups, and mainly around 0.5%. In the supplement, Section 8.4, we list the empirical standard deviations for all months and age groups.

Then, we distribute the expected number of deaths for year $t = 2020, 2021, 2022$ according to the factors $f_{\bar{x},m}$ and $f_{\bar{y},m}$,

$$\mathbb{E}D_{\bar{x},t,m} = f_{\bar{x},m}\mathbb{E}D_{\bar{x},t}, \quad \mathbb{E}D_{\bar{y},t,m} = f_{\bar{y},m}\mathbb{E}D_{\bar{y},t},$$

yielding the expected number of deaths in month m. For $\bar{a}$ a suitable interval in $[0, \infty)$, consistent with the age groups defined by the Federal Statistical Office of Germany, we set

$$\mathbb{E}D_{\bar{a},2021,m} = \sum_{\bar{x}\subset\bar{a}}\mathbb{E}D_{\bar{x},t,m} + \sum_{\bar{y}\subset\bar{a}}\mathbb{E}D_{\bar{y},t,m}.$$

Again, for 2020 we take into account that this is a leap year with one additional day in February. These expected values should be compared to the observed data $d_{\bar{a},t,m}$ for $m = 1, \ldots, 12$. The remarks made at the end of Section 2.1 apply similarly to the computations made in this section.

The monthly mortality probabilities implicitly estimated with this method clearly reflect the excess deaths caused by the usual infections in winter (and possible high temperature weeks in summer), and thus lower mortality probabilities in spring and fall. This is what is expected, and the COVID-19 excess deaths must be compared to these expected mortality waves in winter.

Also note, that we do not assume that the population or the age structure is constant during a year when distributing the expected number of deaths to months. We just assume that the mean population change in the last years is comparable to the situation in 2020-2022, and thus the changes from January to December to January in the last ten years mimic the changes in 2020-2022.

Note that the reported number of deaths in 2022 are from the most current data set of the Federal Statistical Office of Germany: this is still preliminary, and in particular in the second half of 2022, there will still be monor changes within the next weeks and months.

5.2 Results

Following the computations described in the previous section, we calculate the expected number of deaths $\mathbb{E}D_{\bar{a},2021,m}$ for all months $m = 1, \ldots, 12$ in the years $t = 2020, 2021, 2022$.

To compare the expected and the observed values, we again use the relative difference

$$\frac{d_{\bar{a},2021,m} - \mathbb{E}D_{\bar{a},2021,m}}{\mathbb{E}D_{\bar{a},2021,m}}.$$

Table 6: Expected deaths and monthly excess mortality over all age groups.

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	89,441 84,980	-4.99%	90,492 106,803	18.02%	91,328 89,440	-2.07%
m=2	88,627 80,030	-9.70%	86,593 82,191	-5.08%	87,400 82,796	-5.27%
m=3	92,263 87,396	-5.28%	93,345 81,901	-12.26%	94,203 93,719	-0.51%
m=4	81,088 83,830	3.38%	82,022 81,877	-0.18%	82,762 86,179	4.13%
m=5	79,013 75,835	-4.02%	79,895 80,876	1.23%	80,592 81,767	1.46%
m=6	74,508 72,159	-3.15%	75,331 76,836	2.00%	75,979 79,412	4.52%
m=7	78,389 73,795	-5.86%	79,268 76,704	-3.24%	79,960 85,878	7.40%
m=8	76,809 78,742	2.52%	77,661 76,402	-1.62%	78,334 86,359	10.25%
m=9	73,745 74,243	0.68%	74,564 77,931	4.52%	75,208 80,664	7.26%
m=10	80,294 79,781	-0.64%	81,209 85,080	4.77%	81,926 93,881	14.59%
m=11	80,143 85,989	7.30%	81,061 93,915	15.86%	81,779 87,966	7.57%
m=12	87,237 108,792	24.71%	88,266 103,171	16.89%	89,075 113,115	26.99%

In the following sections, we investigate in detail the monthly excess mortality in the age ranges $\bar{a} = [0, 14]$, $[15, 29]$, $[30, 49]$, $[50, 59]$, $[60, 79]$, and $[80, \infty)$. The next figure shows the results for these age groups.

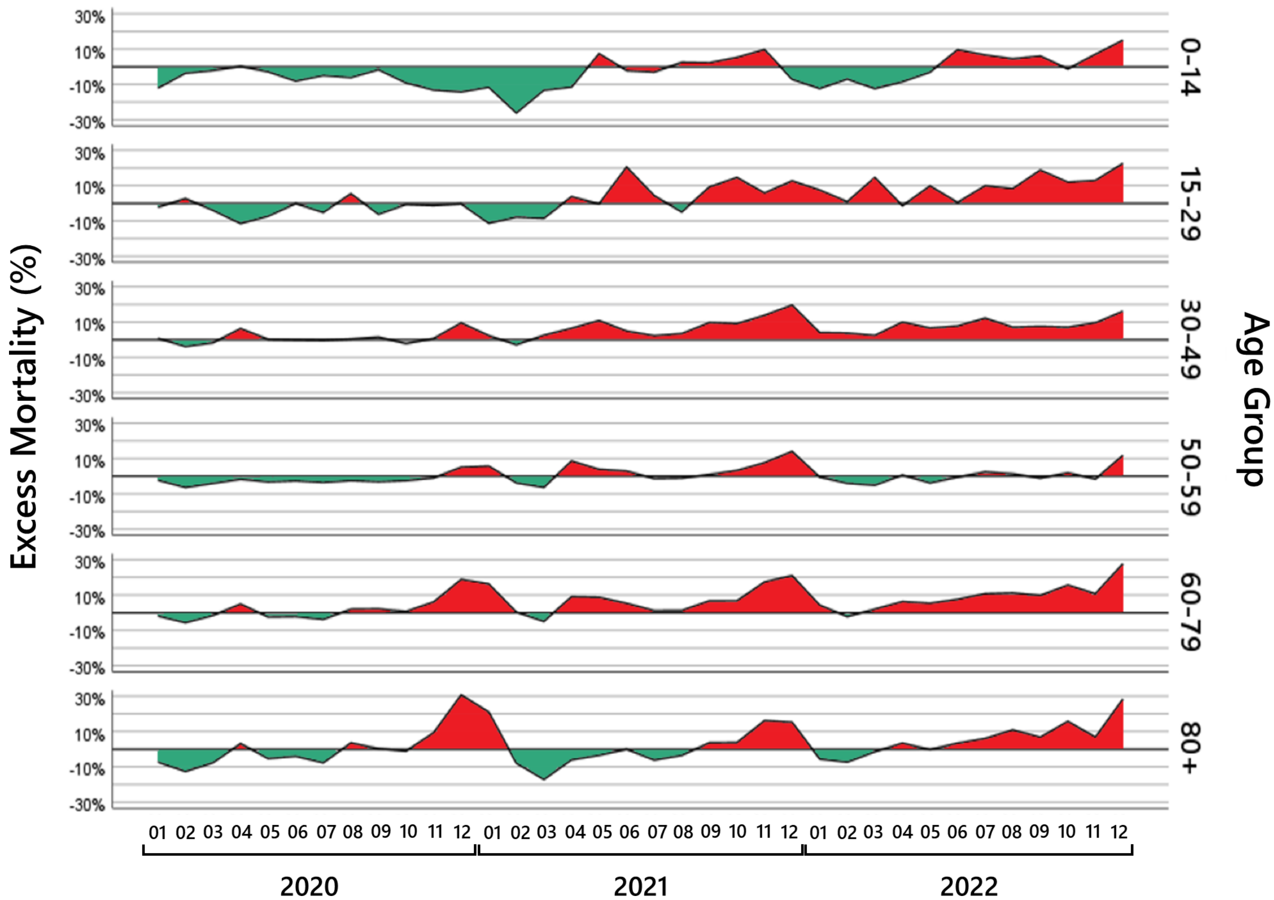

Fig. 8: Development of the monthly excess mortality. For six age groups the black lines show the monthly excess mortality from January 2020 to December 2022. The red-shaded areas show the the time periods where a mortality increase was observed; the green-shaded areas show the time periods where a mortality deficit was observed.

5.2.1 Children $[0, 14]$

In the the age group $[0, 14]$, the number of deaths is small and dominated by the relatively large infant mortality in the first year of life. The expected number of deaths in a month is approximately 300, and hence in the binomial model – which as we know from the investigations in Section 4 heavily under-estimates the standard deviation – we would already expect oscillations at least of the order

$$2\sigma(D_{[0,14],t,m}) \geq 2\sqrt{D_{[0,14],t,m}} \approx 35.$$

Yet such deviations already lead to an excess mortality of more than 10%. The graph in Fig. 8 shows in fact such abrupt oscillations, hence we think that any conclusion relying on these numbers has to be taken with great care. The maybe only notable results are, first, the well accepted fact that children are extremely robust with respect to SARS-CoV-2-infections, and the curve seems to be independent of the usual SARS-CoV-2-infection waves. Second, the presumably different social behavior during the Corona crises seems to lead to a mortality deficit in the younger age groups which is visible here. An exception are the months May and November 2021, and June and November 2022, with a visible positive mortality excess, and December 2022 with a serious mortality excess. In the supplement, Section 8.5, we state the table with precise results.

5.2.2 Young Adults $[15, 29]$

The age group $[15, 29]$ is the first age group where we list the results in detail.

Table 7: Expected deaths and monthly excess mortality $[15, 29]$**.**

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	336 329	-2.22%	327 290	-11.19%	321 345	7.39%
m=2	322 330	2.63%	301 278	-7.73%	296 299	0.87%
m=3	333 320	-3.88%	323 296	-8.39%	318 364	14.51%
m=4	325 288	-11.37%	315 327	3.69%	310 306	-1.38%
m=5	335 311	-7.18%	325 324	-0.36%	320 351	9.72%
m=6	329 329	-0.12%	320 385	20.43%	315 316	0.46%
m=7	353 335	-5.08%	342 357	4.23%	337 370	9.80%
m=8	339 357	5.35%	329 313	-4.83%	324 350	8.17%
m=9	325 305	-6.08%	315 344	9.15%	310 368	18.68%
m=10	322 320	-0.65%	313 358	14.53%	308 344	11.85%
m=11	313 309	-1.22%	304 321	5.74%	299 337	12.83%
m=12	312 311	-0.33%	303 341	12.61%	298 365	22.52%

As for the age group 0-14 the number of expected and observed deaths is small. Hence, again the observed excess mortality has to be interpreted with great care. The numbers until March 2021 are mostly negative and reflect the minimal number of deaths by the two COVID-19 waves in this age range. Somehow unexpectedly, in June 2021, a significant excess mortality is observed, followed by a decrease. However, other than at the beginning of the year, excess mortality remains above zero – with the exceptions of August 2021 and April 2022 – and has visible peaks in October and December 2021, again in March and May 2022 and increases drastically in December 2022.

5.2.3 Adults $[30, 49]$

The age group $[30, 49]$ is the largest group, and we expect approximately 1,800 deaths per month.

Table 8: Expected deaths and monthly excess mortality $[30, 49]$**.**

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	1,949 1,964	0.78%	1,908 1,952	2.29%	1,880 1,954	3.96%
m=2	1,850 1,782	-3.69%	1,750 1,702	-2.75%	1,724 1,788	3.69%
m=3	1,957 1,924	-1.68%	1,917 1,965	2.49%	1,889 1,934	2.37%
m=4	1,816 1,929	6.25%	1,779 1,893	6.41%	1,753 1,926	9.88%
m=5	1,841 1,845	0.24%	1,804 1,998	10.77%	1,778 1,894	6.55%
m=6	1,789 1,788	-0.06%	1,753 1,837	4.82%	1,726 1,859	7.68%
m=7	1,839 1,834	-0.29%	1,802 1,842	2.22%	1,776 1,992	12.19%
m=8	1,814 1,821	0.38%	1,778 1,838	3.39%	1,752 1,875	7.04%
m=9	1,736 1,762	1.48%	1,701 1,865	9.61%	1,677 1,802	7.47%
m=10	1,804 1,766	-2.12%	1,768 1,927	9.02%	1,742 1,864	7.03%
m=11	1,744 1,756	0.67%	1,709 1,944	13.75%	1,684 1,844	9.50%
m=12	1,831 2,004	9.46%	1,794 2,144	19.52%	1,768 2,051	16.04%

As in the age group $[15, 29]$, the numbers in year 2020 are mostly unremarkable, and reflect the minimal number of deaths by the first COVID-19 wave in April 2020, and a visible excess mortality in December 2020 in this age range. Then, the excess mortality fluctuates around zero until March 2021. From an actuarial perspective, we would expect this to continue until winter.

Somehow unexpectedly, in April and mainly in May 2021, a significant increase in excess mortality is observed, occurring one month before the similar excess mortality in the age group $[15, 29]$. The excess mortality in May is followed by a decrease up to August. However, other than at the beginning of the year, excess mortality remains above zero so that the increase in excess mortality in April and May is not compensated for. In September, there is again a significant excess mortality which increases in November and reaches 20% in December 2021. In 2022 the excess mortality stays always positive, fluctuating around 8%, and reaches again a serious excess mortality of 16% in December.

5.2.4 The exceptional age group $[50, 59]$

The age group $[50, 59]$ seems to be exceptional resilient against the factors that drive the excess mortality in the other age groups. There are neither huge peaks of mortality excess nor serious mortality deficits, the mortality excess fluctuates around zero.

The numbers in year 2020 are close to zero, ignoring the first COVID-19 wave in April 2020, and show a some mild excess mortality in winter 2020 in this age range. There is a visible peak in April 2021 and December 2021. In 2022, the excess mortality is always close to zero, only in December there is some positive excess mortality.

This leads to the surprising result, visualized in Fig. 5, that in all pandemic years 2020 to 2022 this age goup has – in contrast to all neighboring age groups – no serious excess mortality. In the supplement, Section 8.5, we state the table with precise results.

5.2.5 The retirement age $[60, 79]$

This group consists of the ages $[60, 79]$, a 'mixed' group where parts of this population are still in health, and parts are already vulnerable, and for these a SARS-CoV-2-infection can be dangerous. This is visible in the results for 2020.

Table 10: Expected deaths and monthly excess mortality [60, 79]**.**

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	28,409 27,905	-1.77%	27,857 32,372	16.21%	27,627 28,787	4.20%
m=2	27,910 26,369	-5.52%	26,423 26,505	0.31%	26,205 25,643	-2.15%
m=3	29,147 28,708	-1.51%	28,569 27,195	-4.81%	28,326 28,883	1.97%
m=4	26,058 27,314	4.82%	25,555 27,839	8.94%	25,347 26,916	6.19%
m=5	25,811 25,201	-2.36%	25,320 27,507	8.64%	25,119 26,443	5.27%
m=6	24,481 23,960	-2.13%	24,022 25,274	5.21%	23,836 25,620	7.49%
m=7	25,625 24,683	-3.68%	25,148 25,440	1.16%	24,954 27,620	10.68%
m=8	25,105 25,595	1.95%	24,631 24,939	1.25%	24,437 27,153	11.11%
m=9	24,060 24,568	2.11%	23,603 25,164	6.62%	23,415 25,690	9.71%
m=10	25,923 26,101	0.69%	25,422 27,119	6.67%	25,216 29,139	15.56%
m=11	25,669 27,211	6.01%	25,164 29,519	17.31%	24,954 27,621	10.69%
m=12	27,623 32,802	18.75%	27,076 32,747	20.95%	26,848 34,290	27.72%

The results show a decent peak for April 2020, followed by a significant peak around December 2020. The peak of December 2020 continues in January 2021, but then turns into a mortality deficit. In April 2021 we observe a serious excess mortality for two months. In September and October 2021 we see a decent, and in November and December 2021 again a significant excess mortality. The year 2022 starts with an unremarkable mortality deficit which again in April turns into an excess mortality which remains for the rest of the year on a high level, and is even above 27% in December.

5.2.6 Old ages $[80, \infty)$

The last group consists of the ages ≥ 80 (beyond the expected life time in Germany which is approximately at the age of 80), where large parts of the vulnerable population belong to, and a SARS-CoV-2-infection is particularly dangerous. We list the total expected monthly number of deaths $\mathbb{E}D_{\bar{a},t,m}$ for the population above the age of the average life expectancy, $\bar{a} = [80, \infty)$, the observed number of deaths and the relative difference.

Table 11: Expected deaths and monthly excess mortality $[80, \infty)$**.**

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	53,222 49,408	-7.17%	54,945 66,478	20.99%	56,154 53,070	-5.49%
m=2	53,232 46,559	-12.54%	53,052 48,891	-7.84%	54,210 50,303	-7.21%
m=3	55,264 51,095	-7.54%	57,043 47,315	-17.05%	58,287 57,447	-1.44%
m=4	47,779 49,267	3.12%	49,324 46,397	-5.93%	50,404 52,079	3.32%
m=5	45,848 43,459	-5.21%	47,332 45,728	-3.39%	48,365 48,251	-0.23%
m=6	42,944 41,260	-3.92%	44,334 44,312	-0.05%	45,299 46,808	3.33%
m=7	45,469 42,018	-7.59%	46,936 44,096	-6.05%	47,955 50,826	5.99%
m=8	44,518 46,067	3.48%	45,954 44,384	-3.42%	46,952 52,037	10.83%
m=9	42,744 42,870	0.30%	44,125 45,689	3.54%	45,083 48,112	6.72%
m=10	47,087 46,579	-1.08%	48,612 50,414	3.71%	49,669 57,455	15.68%
m=11	47,343 51,720	9.25%	48,873 56,735	16.09%	49,932 53,308	6.76%
m=12	52,200 68,197	30.65%	53,887 62,069	15.18%	55,059 70,696	28.40%

The results for the age group $[80, \infty)$ show a decent peak for April 2020, and a huge peak around December 2020. The peak of December 2020 continues in January 2021, but then turns into a mortality deficit until April 2021, where the downwards trend stops. In September and October, we see a decent, and in November and December 2021 again a serious excess mortality. The year 2022 starts with a mortality deficit which again in June turns into an excess mortality which remains on a high level for the rest of the year, and reaches a extremum in December with more than 28% excess mortality.

Although the trend in the age groups $[60, 79]$ and $[80, \infty)$ looks parallel, it is interesting to point out the differences. As can be seen in in Fig. 8, the curve for the age group $[80, \infty)$ is below and somehow parallel to the curve for the age group $[60, 79]$. The main difference is the deviation of the age group $[60, 79]$ in April and May 2021 where a jump in the mortality behaviour for the this age group is visible. The age group $[80, \infty)$ seems to be more resistant to mortality causes at a larger scale than other age groups. At certain moments, some people die some months before or after the 'expected' time of death, but the curve for the excess mortality mostly oscillates around the 0% axes. A visible mortality deficit until October 2020 with exceptions in April and August is followed by a huge mortality excess peak at the turn of the year 2020/2021. This in turn is more or less compensated by the mortality deficit in January to July 2021, the peak around November and December 2021 is nearly compensated in February to March 2022. Something surprisingly, since August 2022 the mortality excess stays continuously on a very high level and even increases again to nearly 30% in December 2022.

It is interesting to make this observations visible by calculating the cumulative excess mortality since January 2020 in absolute numbers. Maybe due to a comparably mild flue saison in 2019/2020, we start with a negative value. In July 2020, up to $20,000$ people more than expected are still alive, which is compensated in December 2020 to February 2021, where the curve is $10,000$ above the expectation, and then the curve fluctuates to $-10,000$, to $+10,000$, and until July 2022 where it is approximately $7,000$. This shows that a mortality deficit or a excess mortality in the age group $[80, \infty)$ usually just shifts the time of death by some months. This changes in the last months of 2022 where where we see a cumulated excess mortality of $42,000$ deaths at the end of the year.

This is in contrast to the situation for the age group $[60, 79]$. The cumulative excess mortality is steadily increasing up to $55,000$ deaths at the end of year 2022. The difference between both age groups is demonstrated in Fig. 9.

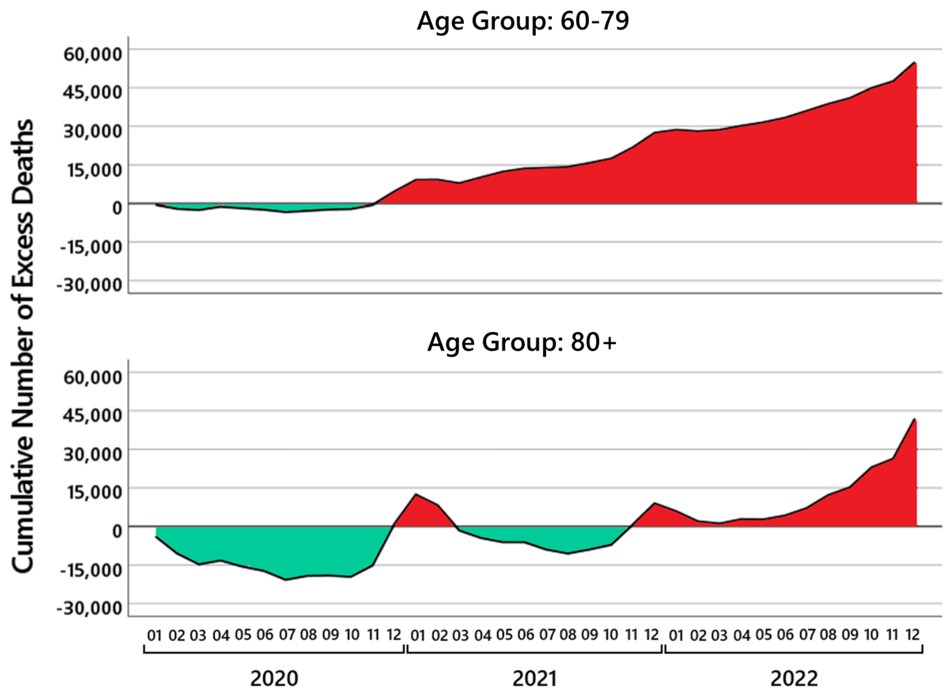

Fig. 9: The cumulative excess mortality. The green areas show the regions of a cumulative mortality deficit, the red areas of a cumulative excess mortality from January 2020 to December 2022. The age group $[80, \infty)$ is oscillating, the age group $[60, 79]$ nearly monotone increasing.

5.3 Stillbirths in the years 2019 to 2022 in Germany

In all previous studies on excess mortality during the COVID-19 pandemic, only the increase in mortality for the age groups 0 and above have been examined. In the following, it is examined whether similar increases in mortality than that found for the age groups 0 and above are also found at the level of stillbirths.

One problem with analyzing excess mortality at the level of stillbirths in Germany is that the definition of a ‘stillbirth’ has been changed at the end of 2018. Up to this point, a stillborn child was considered a stillbirth if a birth weight of at least 500 grams was reached. Since the end of 2018, a stillborn child is considered a stillbirth if at least 500 grams or the 24th week of pregnancy was reached, which led to a diagnostically related increase in stillbirths. This means that the figures on stillbirths are only validly comparable from 2019 onwards. Thus, estimating excess mortality at the level of stillbirths based on a modeling of long-term trends in mortality is problematic. Furthermore, the empirical standard deviation occurred in the years before cannot be determined. Due to these reasons, we only descriptively report the course of stillbirths from 2019 on.

When analyzing the number of stillbirths, it is important to note that they must be placed in relation to the number of total births, because an increase or decrease in the number of total births is automatically accompanied by an increase or decrease in stillbirths. Fig. 10 shows in the first panel the number of live births per quarter [20] and in the second panel the number of stillbirths per quarter [21] since 2019. As can be seen from the shift in the seasonal peaks of the stillbirths compared to the seasonal peaks of the live births, stillbirths precede live births from the same pregnancy cohort by about one trimester. Thus, to correctly control for the effect of a general increase or decrease in the number of total births, the number of total births must be calculated as the sum of the number of stillbirths in a quarter and the number of live births in the following quarter.

Fig. 10 shows in the third panel the number of stillbirths per 1,000 total births, and in the fourth panel the quarterly increase in the number of stillbirths per 1,000 total births in the years 2021, and 2022 compared to the mean across the years 2019 and 2020. Note that the number of stillbirths per 1,000 total births cannot be determined for the third quarter 2022 because the number of live births in the fourth quarter 2022 has not yet been published by the Federal Statistical Office of Germany. Also note that the quarterly pattern observed in the year 2022 cannot be validly interpreted because only preliminary data is available based on the reporting month, with the data being assigned to the month of death only with the publishing of the final data by the Federal Statistical Office of Germany.

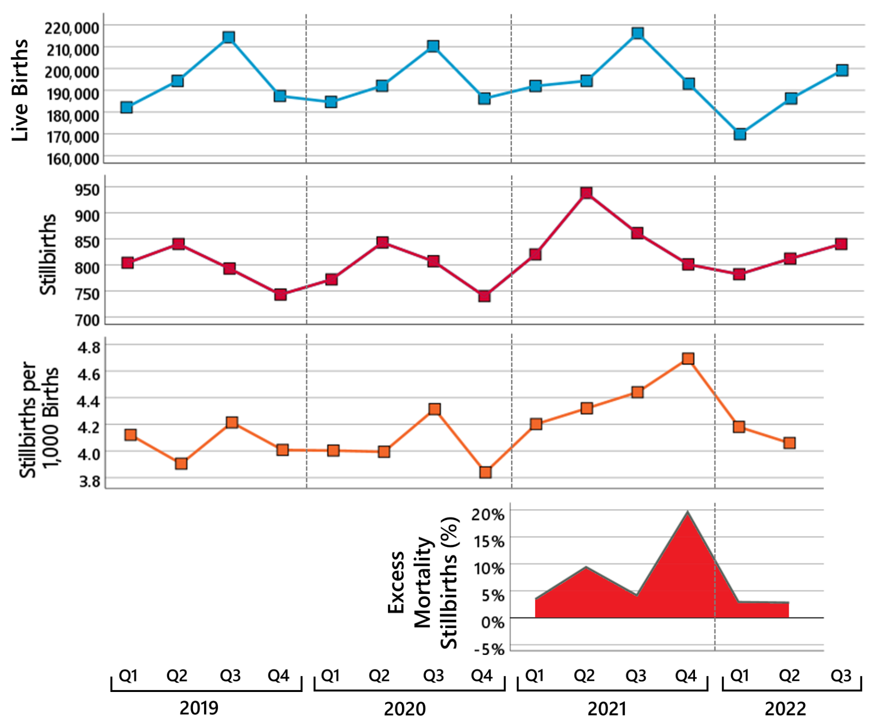

Fig. 10: Stillbirths in the years 2019 to 2022 in Germany. The first panel shows the number of live births per quarter from 2019 to 2022, the second panel the number of stillbirths per quarter from 2019 to 2022, the third panel the number of stillbirths per 1,000 total births (sum of the number of stillbirths in a quarter and the number of live births in the following quarter) per quarter from 2019 to 2022, and fourth panel the quarterly increase in

the number of stillbirths per 1,000 total births in the years 2021, and 2022 compared to the mean across the years 2019 and 2020.

Until the end of 2021, the number of live births shows a stable course with a regularly repeating seasonal pattern. In the first quarter 2022, a sudden and sustained drop in the number of births is observed. Regarding the number of stillbirths, a stable course is observed until the end of the first quarter of 2021. In the second quarter of 2021, a sudden increase in stillbirths is observed, despite the stable course of live births until the end of 2021. Compared to the quarterly number of stillbirths per 1,000 total births in the years 2019 and 2020, the number of stillbirths increased by 9.4% in the second quarter of 2021 and by 19.6% in the fourth quarter of 2021. This is similar to the increases in mortality observed for the age groups 0 and above: whereas in the year 2020 no change in stillbirths is found compared to the previous year, in the year 2021, a sudden increase in stillbirths is observed in the second quarter of 2021 which reaches a high level in fourth quarter of 2021.

6 Discussion

In the previous sections, we estimated the expected number of all-cause deaths and the increase in all-cause mortality for the pandemic years 2020 to 2022 in Germany. The results revealed several previously unknown mortality dynamics that require a reassessment of the mortality burden brought about by the COVID-19 pandemic.

The analysis of the yearly excess mortality showed a marked difference between the pandemic years 2020, 2021, and 2022. Whereas in the year 2020 the observed number of deaths was extremely close to the expected number with respect to the empirical standard deviation, in 2021, the observed number of deaths was far above the expected number (more than twice the empirical standard deviation), and further increases in 2022 (above four times the standard deviation). An age-dependent analysis showed that the strong excess mortality observed in 2021, and 2022 was mainly due to an above-average increase in deaths in the age groups between 15 and 79. A detailed analysis of the monthly excess mortality showed that the high excess mortality observed in the age groups between 15 and 79 started to accumulate from April 2021 onwards. A similar pattern was observed for the number of stillbirths which was similar to the previous years until March 2021, after which also a sudden and sustained increase was observed.

Taken together, these findings raise the question what happened in spring 2021 that led to a sudden and sustained increase in mortality, although no such effects on mortality had been observed during the COVID pandemic so far. In the following sections, possible explanatory factors are explored.

6.1 Possible factors influencing mortality

As already mentioned, apart from the population structure, the number of deaths in a year depends on several different factors, the most important being maybe the severity of the flue, and the number of extremely hot weeks. The fluctuations between different years, and thus the approximation of the empirical standard deviation $\hat{\sigma}(D_t)$ in Section 4, includes all these factors. It is unclear, rather subjective, and most probably impossible to precisely define 'extreme events', to calculate the influence of such extreme events, and to adjust mortality to 'entirely normal' years. Thus our calculations gives the expected number of deaths taking into account all these extreme and non-extreme effects which are contained in the different life tables. We tried to quantify the sensitivity of our approach in Section 3 and Section 4 against the background of extreme events in the last years.

For the pandemic years 2020 to 2022, it is clear that the number of deaths has been influenced directly and indirectly by COVID-19. First, clearly, there has been a serious number of COVID-19 deaths, either as the only reason for death or in combination with several other causes, which also might have caused death independently of COVID-19, see e.g. the discussion in Section 5.2.6 and in the forthcoming Section 6.2. Second, the vaccination campaign which started in 2021 should be visible in a reduced excess mortality, or even better as a mortality deficit. An attempt to compare our results to the number of vaccinations is the content of Section 6.3.

Third, the indirect effects on mortality due to the COVID-19 measures are extremely harder to quantify. Several aspects may contribute to an excess mortality or a mortality deficit. In Germany, strict control measures since 2020 limited personal freedom, schools were partially closed, there were severe lockdowns. This substantially influenced the risk of road accidents and other outdoors casualties. On the other hand, many clinical services have been delayed or avoided in 2020, 2021, and 2022. All these and many more factors influenced mortality in different directions and on different time scales, but most of them are hard to measure, many effects are highly correlated, and it seems to be impossible to quantify the overall impact of the control measures on the number of deaths.

6.2 COVID-19 deaths and mortality

In this section we compare the excess mortality since March 2020 to the reported number of COVID-19 deaths by the German Robert Koch Institute. The Robert Koch Institute provides the weekly number of COVID-19 deaths [22] for the age groups $[0, 9]$, $[10, 19]$, … (which differ from the age groups used by the Federal Statistical Office of Germany); in addition these numbers are incomplete because all numbers below 4 are not stated due to data security reasons.

Even when the reporting system in Germany seems to be imprecise and partially insufficient, there should be a serious correlation between the reported number of deaths and the excess mortality. To make the difference between the excess deaths and the COVID-19 deaths visible, we show the monthly development of the number of reported COVID-19 deaths and the excess mortality in Fig. 11, and on the same scale the difference between both in Fig. 12.

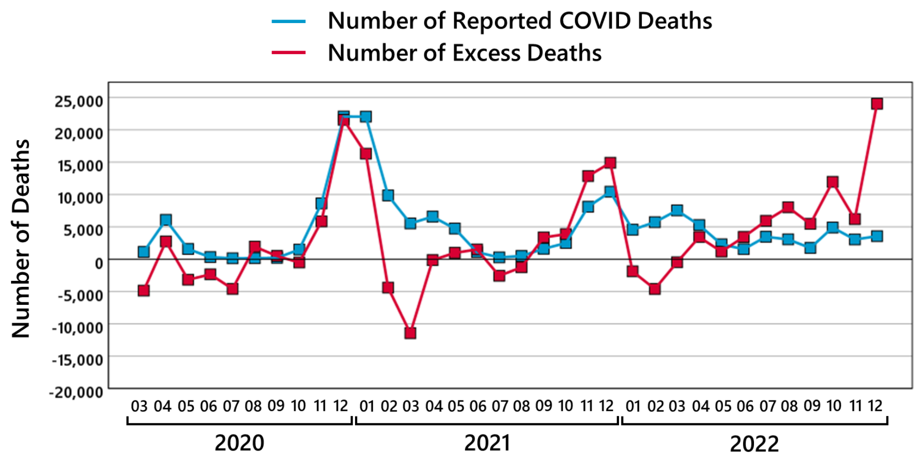

Fig. 11: COVID-19 deaths versus excess mortality. The blue squares show the number of reported COVID-19 deaths, the red squares the mortality deficit, respectively the excess mortality from January 2020 to December 2022.

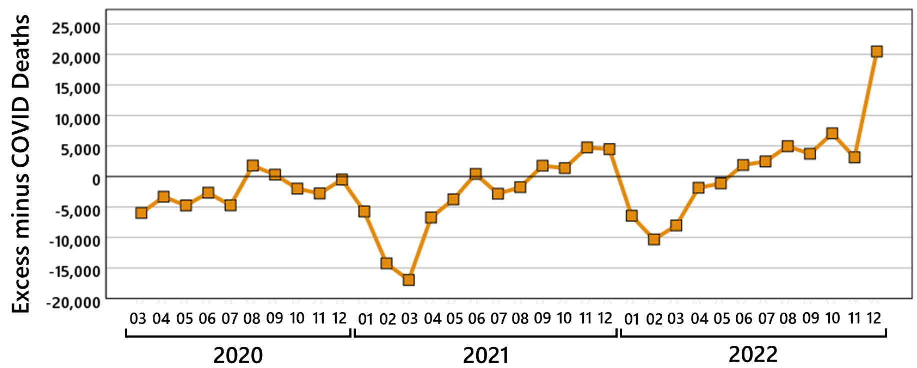

Fig. 12: Difference of COVID-19 deaths and excess mortality. The line shows the difference between the number of excess deaths and the number of COVID-19 deaths from January 2020 to December 2022.

Until July 2020, the number of excess deaths is below the number of reported COVID-19 deaths, and except for April 2020, a mortality deficit is observed despite the reporting of COVID-19 deaths. From August 2020 to December 2020, the numbers of excess deaths and reported COVID-19 deaths largely coincide. However, after that, the number of COVID-19 deaths stays on a high level while all-cause mortality decreases, and in February and March 2021, a noticeable all-cause mortality deficit is observed despite a high number of reported COVID-19 deaths of up to $10,000$. Starting in September 2021, a marked increase in excess mortality is observed that is not accompanied by a comparable increase in reported COVID-19 deaths. From January 2022 onwards, both curves decouple, showing partly opposite patterns of increases versus decreases. From June 2022 onwards, the number of excess deaths is increasingly larger than the number of reported COVID-19 deaths, reaching very large differences in December 2022 where over 24,000 excess deaths are observed but only slightly more than 3,500 COVID-19 deaths reported. It thus is obvious that the number of reported COVID-19 deaths is fluctating somehow independently of the excess mortality, and contains a large number of 'expected' deaths.

It is also elucidating to compare the cumulative number of COVID-19 deaths to the cumulative number of excess deaths in Fig. 13. The cumulative number of reported COVID-19 deaths is increasingly higher than the cumulative number of excess deaths.

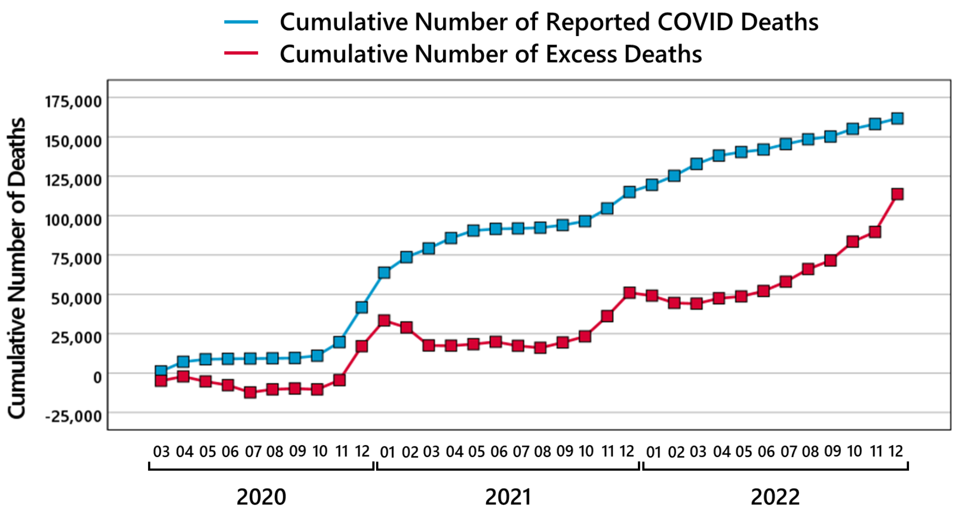

Fig. 13: Cumulative COVID-19 deaths versus cumulative excess mortality. The blue squares show the cumulative number of reported COVID-19 deaths, the blue squares the cumulative mortality deficit, respectively the excess mortality from March 2020 to December 2022.

Because the Federal Statistical Office of Germany uses different age groups, direct comparisons are made difficult, e.g. for the age group $[15, 29]$ used in the previous section. Therefore we divide the number of COVID-19 deaths in the age group $[10, 19]$ into two equal parts to obtain the number of COVID-19 deaths in the age groups $[0, 14]$ and $[15, 29]$, estimate the number of deaths for those weeks with less than 4 deaths, and divide each week where two months overlap between these two months.

We list the number of excess deaths in six age groups and compare these to the COVID-19 deaths, as a timetable we use the first pandemic year 04/2020 – 03/2021 and compare this to the second year 04/2021 – 03/2022. It seems difficult to find a convincing pattern which explains the dependence of the excess deaths on the COVID-19 deaths.

Table 12: Expected vs. observed deaths, and excess deaths vs. COVID-19 deaths

	04/20–03/21		04/21–03/22	
age range	exp. obs.	abs.diff. COVID	exp. obs.	abs.diff. COVID
0-14	3,519 3,195	-324 25	3,514 3,425	-89 73
15-29	3,904 3,729	-175 81	3,801 4,078	277 120
30-49	21,790 22,124	334 617	21,380 22,964	1,584 1,256
50-59	58,269 57,385	-884 2,094	57,383 58,775	1,392 3,072
60-79	313,204 323,507	10,303 21,351	308,100 328,861	20,761 18,645
80-∞	580,971 594,121	13,150 53,882	598,030 600,644	2,614 30,465
0-∞	981,656 1,004,061	22,405 78,050	992,209 1,018,747	26,538 53,631

In the age groups $[0, 14]$, $[15, 29]$ and $[50, 59]$, mortality deficits occur but the number of COVID-19 deaths is positive. In the age group $[30, 49]$, the number of excess deaths seems to fit the number of COVID-19 deaths. In the age group between 60 and 79 years, of the approximately $40,000$ COVID-19 deaths reported until the end of March 2022, approximately $9,000$ did not show up as excess deaths. The strongest divergence is found in the age group over 80 years, where of the approximately $84,000$ COVID-19 deaths reported until the end of March 2022, approximately $69,000$ did not show up as excess deaths. Taken together, of the $132,000$ reported COVID-19 deaths in the age groups over 20 years, more than $83,000$ did not show up as excess deaths and are thus contained in the 'expected' number of deaths. Since other factors beyond COVID-19, such as delayed or avoided clinical services, may have contributed to the number of excess deaths, the number of reported COVID-19 deaths which actually represent expected deaths and not excess deaths is probably even higher.

Taken together, it seems to be misleading to measure the risk of the COVID-19 pandemic only using the reported deaths. One should rather use the excess mortality curve than the number of reported COVID-19 deaths, or a combination of both, to carve out the moments of high risk, and to evaluate the total risk of a pandemic.

Beyond the problem that the number of reported COVID-19 deaths cannot be validly used to assess the effects of the COVID-19 pandemic on mortality, it seems also unlikely that the high excess mortality in 2021 in the age groups under 80 years can be explained by COVID-19 deaths because the marked increases in excess mortality in April to June 2021 - the mortality increases abruptly by 13% from March to April 2021 in the age group between 15 and 59 - and also in October to December 2021 were not accompanied by comparable increased in the number of COVID-19 deaths. Furthermore, it seems also very unlikely that the abrupt increase of the mortality is due to delayed or avoided clinical services which should lead to much smoother changes, or due to side effects of COVID-19 measures. This is the more unlikely in the year 2022 where excess mortality increases even further despite a decrease in reported COVID-19 deaths, and despite the fact that clinical care should slowly return to normal. Thus, it remains to investigate the factors which could lead to the surprising jumps in excess mortality in April to June 2021, in October and November 2021, and in the year 2022.

6.3 COVID-19 vaccination and mortality

In April 2021, an extensive vaccination campaign started in Germany. Comparing the number of vaccinations [23] to the excess mortality should show the sum of two theoretically possible effects of vaccinations: the prevention of infections and deaths through immunization should decrease the number of excess deaths, and, if there were side effects in the form of deaths, the occurrence of such side effects should increase the number of excess deaths. The following Fig. 14 shows on the left scale the number of excess deaths, respectively death deficit, and on the right scale the number of vaccinations.

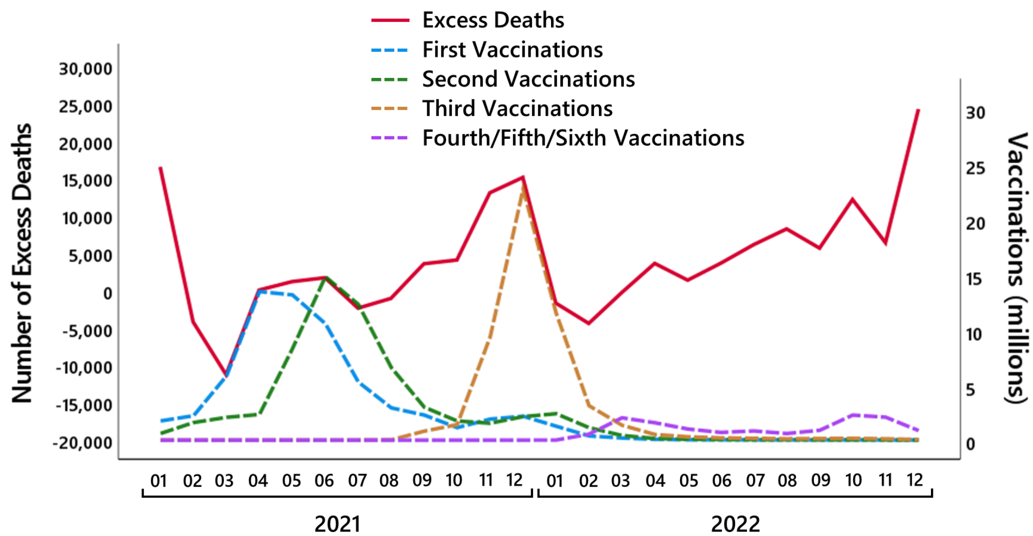

Fig. 14: Number of vaccinations versus excess mortality. The red line shows the death deficit, respectively the excess deaths, the four dashed lines the number of vaccinations from January 2021 to June 2022.

As can be seen in Fig. 14, a visible positive effect of the vaccinations on excess mortality does not occur. Instead, the opposite is observed. Although at the beginning of September 2021, 82.7% of the population over 60 years and 65.2% of the population from 18–59 years were fully vaccinated, the number of excess deaths nevertheless started to increase strongly, reaching a level of almost $15,000$ excess deaths in December 2021, where 86.1% of the population over 60 years and 75.7% of the population from 18–59 years were fully vaccinated. At the beginning of March 2022, 88.6% of the population over 60 years and 83.3% of the population from 18–59 years were fully vaccinated, and 77.4% of the population over 60 years and 60.6% of the population from 18–59 years had even received a third vaccination. Despite this high level of population-wide vaccinations, excess mortality starts to monotonically increase, reaching a level of more than $24,000$ excess deaths in December 2022.

The observation that excess mortality increased with increased vaccinations not only casts some doubts on the effectiveness of the vaccinations. From the perspective of pharmacovigilance, such an observation represents a safety signal because such a temporal relationship between the number of excess deaths and the number of vaccinations would occur if the vaccinations caused unwanted deaths as a side effect. This impression is strengthened by a closer inspection of the courses of the numbers of vaccinations and excess deaths in the second pandemic year April 2021 to March 2022. The number of excess deaths closely follows the course of the number of vaccinations, showing an increase as soon as the number of vaccinations increases, and a decrease as soon as the number of vaccinations decreases. A similar safety signal in terms of a temporal relationship between an increase in vaccinations and deaths is also observed at the level of stillbirths. Exactly with the beginning of the vaccinations in the age group $[18, 59]$, the number of stillbirths suddenly increased after being stable for at least the two previous years.

Safety signals such as the observation of a temporal relationship between the administration of vaccines and the occurrence of adverse events do not necessarily imply a causal relationship since there may be potential third variables that influence both the course of vaccinations and the course of excess deaths. Thus, a safety signal does not indicate a causal relationship between a side effect and a drug but is only a hypothesis that calls for further assessment.

However, there is first evidence from autopsy studies that vaccinations can at least cause deaths. For instance, in a study of a research team led by Peter Schirmacher [24], out of 35 bodies found unexpectedly dead at home with unclear causes of deaths within 20 days following COVID-vaccination, autopsies revealed causes of death due to pre-existing illnesses in only 10 cases. From the remaining 25 cases, in three cases it was concluded from the autopsies that vaccination-induced myocarditis was the likely cause of death, and in two cases it was concluded that this was possibly the case. According to [24], Supplementary Table 1, vaccination was the cause of deaths in further cases as well. For instance, a 38-year-old man with no relevant preexisting disease died due to vaccine-induced thrombotic thrombocytopenia; a 23-year- old woman with no relevant preexisting disease died due to pulmonary embolism, which may also suggest vaccination as the cause of death.

These findings indicate two important aspects. First, the findings show that COVID-vaccinations can cause deaths as a side effect. Second, the findings show that deadly side effects of COVID-vaccinations are not extreme exceptional cases. The authors of the paper correctly conclude that epidemiological conclusions in terms of incidence or risk estimation cannot validly drawn from their study. However, the fact that the re-examination of only 35 deaths of only one specific type (bodies found unexpectedly dead at home) in only a small region in Germany (catchment area of the Heidelberg University Hospital) already reveals so many deaths that have likely or probably been caused by a COVID-vaccination at least suggests that COVID-vaccine-induced deaths are not extremely unlikely.

Given the temporal relationship between the increase in vaccinations and excess mortality from the beginning of the vaccinations campaign onwards, it seems surprising that a respective safety signal has not been detected in the pharmacovigilance by the Paul-Ehrlich-Institut (PEI), which is responsible for the safety monitoring of drugs in Germany. A closer inspection of the methods used by the PEI to monitor possibly deadly side effects of the COVID-vaccinations reveals that a flawed safety analysis is used that will not indicate a safety signal even if a vaccine causes extremely large numbers of unexpected deaths.

The PEI uses a so-called observed-versus-expected analyses where the expected number of all-cause deaths in the vaccinated group is compared to the number of deaths that is has been reported to the PEI with a suspected connection to a COVID-vaccination. If the number of reported suspected vaccine-related deaths is not significantly higher than the number of expected all-cause deaths, the PEI concludes that there is no safety problem. For instance, in the safety report of August 19, 2021 [25], the PEI calculates that $75,284$ all-cause deaths within a period of 30 days after the vaccinations are expected for the group of people that have been vaccinated with Corminaty. From the fact that the number of 926 reported suspected vaccine-related deaths does not exceed the threshold of the expected $75,284$ all-cause deaths, the PEI concludes that there is no warning signal of increased post-vaccination mortality for Corminaty.

Such a safety analysis is profoundly absurd. For a safety signal to occur, more suspected vaccine-related deaths would need to be reported than are caused by all other causes of death (cancer, heart disease, stroke, etc.) combined. Thus, it is not surprising that a safety signal has not been detected in the pharmacovigilance by the PEI, because the occurrence of safety signals is essentially impossible due to the used flawed methods.

Since the available mortality data do not allow to determine the expected and observed numbers of deaths for the vaccinated group only, it is impossible to examine what would have been observed if the PEI had applied a correct safety analysis. However, to at least demonstrate how a proper observed-versus-expected analysis should be done, two time periods can be compared: the time of April 2020 to March 2021 (first pandemic year) can be used as a rough estimate of the expected number of excess deaths without vaccinations. The estimated expected excess deaths can be compared with the observed number of excess deaths in the time of April 2021 to March 2022 (second pandemic year) where large parts of the population were vaccinated. The following Table 13 shows the results of such an analysis for six age groups.

Table 13: Expected deaths and excess mortality 04/20–03/22.

	04/20–03/21			04/21–03/22		
age range	exp. obs.	abs.diff.	rel.diff.	exp. obs.	abs.diff.	rel.diff.
0-14	3,519 3,195	-324	-9.20%	3,514 3,425	-89	-2.54%
15-29	3,904 3,729	-175	-4.48%	3,801 4,078	277	7.28%
30-49	21,790 22,124	334	1.53%	21,380 22,964	**1,584**	7.41%
50-59	58,269 57,385	-884	-1.52%	57,383 58,775	1,392	2.43%
60-79	313,204 323,507	10,303	3.29%	308,100 328,861	**20,761**	6.74%
80-∞	580,971 594,121	13,150	2.26%	598,030 600,644	2,614	0.44%

For alle age groups under eighty years, a significant mortality increase is observed in the second pandemic year where large parts of the population were vaccinated. According to the empirical standard deviations for the different age groups given in Section (8.3), $\hat{\sigma}(d_{[0,14]}) = 158$, $\hat{\sigma}(d_{[15,29]}) = 148$, $\hat{\sigma}(d_{[30,49]}) = 427$, $\hat{\sigma}(d_{[50,59]}) = 868$, $\hat{\sigma}(d_{[60,79]}) = 5,088$ and $\hat{\sigma}(d_{[80,\infty)}) = 9,924$, excess mortalities which are far beyond $2\hat{\sigma}$ did not occur in the in the first pandemic year without vaccinations but only in the second pandemic with vaccinations (age groups $[30, 49]$ and $[60, 79]$; these are highlighted in the table). Thus, the amount of excess mortality observed in the second pandemic year with vaccinations is much higher than the amount of excess mortality in the first pandemic year without vaccinations. This on the one hand contrasts with the expectation that the vaccination should decrease the number of COVID-19 deaths, and on the other hand indicates a safety signal.

The only exception is the last age group $[80, \infty)$, where in the first year a larger number of excess deaths was observed than in the second year. However, when interpreting this finding, it has to be taken into account that in this age range, there was a huge mortality deficit from 2019 until October 2020, which was compensated in November, December 2020, and January 2021. Such an effect could not occur a second time within one year. Furthermore, even if the mortality decrease from the first to the second pandemic year observed in the age group $[80, \infty)$ would be a direct effect of the vaccinations, this would, at least according to the results in the Table 13, not justify the vaccination of the whole population independently of age. In total, the decrease of the number of excess deaths by $10,550$ in the age groups above 80 and the increase of the number of deaths by $14,657$ in the younger age ranges yield a negative net effect.

Taken together, although one would expect that vaccinating large parts of the population should have reduced excess mortality, the contrary is observed. Both excess mortality and the number of stillbirths increased with increased vaccinations, and excess mortality was in all age groups below 80 years higher in the second year of the pandemic, where large parts of the population were vaccinated, than in the first year, where almost nobody was vaccinated. These observations are surprising and should lead to several more detailed investigations from different scientific fields to rule out that these safety signals occur due to the existence of unknown side effects of the COVID-vaccines.

7 Conclusion

The present study used the state-of-the-art method of actuarial science to estimate the expected number of all-cause deaths and the increase in all-cause mortality for the pandemic years 2020 to 2022 in Germany.

In 2020 the observed number of deaths was extremely close to the expected number, but in 2021 the observed number of deaths was far above the expected number in the order of twice the empirical standard deviation, and in 2022 above the expected number even more than four times the empirical standard deviation. The analysis of the age-dependent monthly excess mortality showed, that a high excess mortality observed in the age groups between 15 and 79 starting from April 2021 is responsible for the excess mortality in 2021, and 2022. An analysis of the number of stillbirths revealed a similar mortality pattern than observed for the age group between 15 and 79 years.

As a starting point for further investigations explaining this mortality patterns, we compared the excess mortality to the number of reported COVID-19 deaths and the number of COVID-19 vaccinations. This leads to several open questions, the most important being the covariation between the excess mortality and the COVID-19 vaccinations.

References

[1] von Stillfried S., Bülow R. D., Röhrig R., Boor P., for the German Registry of COVID-19 Autopsies (DeRegCOVID): First report from the German COVID-19 autopsy registry. *The Lancet Regional Health - Europe* **15**, 100330 (2022). doi.org/10.1016/j.lanepe.2022.100330

[2] Saragih I. D., Advani S., Saragih I. S., Suarilah I., Susanto I., Lin C. J.: Frailty as a mortality predictor in older adults with COVID-19: A systematic review and meta-analysis of cohort studies. *Geriatric Nurs.* **42**(5), 983–992 (2021). doi.org/10.1016/j.gerinurse.2021.06.003

[3] Hung I. F. N., Zhang A. J., To K. K. W., Chan, J. F. W. and Zhu S. H. S., Zhang R., Chan T., Chan K., Yuen, K.: Unexpectedly Higher Morbidity and Mortality of Hospitalized Elderly Patients Associated with Rhinovirus Compared with Influenza Virus Respiratory Tract Infection. *Int. J. Mol. Sci.* **18**, 259 (2017). doi.org/10.3390/ijms18020259

[4] Baum K.: Considerations on excess mortality in Germany in the year 2020 and 2021. *Dtsch. Med. Wochenschr.* **147**(7), 430–434 (2022).

[5] COVID-19 Excess Mortality Collaborators: Estimating excess mortality due to the COVID-19 pandemic: a systematic analysis of COVID-19-related mortality, 2020–21. *The Lancet* **399**, 1513–1536 (2022). www.thelancet.com/action/showPdf?pii=S0140-6736%2821%2902796-3

[6] Bauer, R., Speringer, M., Frühwirt, P., Seidl, R., Trautinger, F.: Assessing excess mortality in Vienna and Austria after the first year of the COVID-19 pandemic. *Vienna Yearbook of Population Research* **20**, 1–22, (2022). https://doi.org/10.1553/populationyearbook2022.dat.1

[7] De Nicola, G., Kauermann, G., Höhle, G.: On assessing excess mortality in Germany during the COVID-19 pandemic (Zur Berechnung der Übersterblichkeit in Deutschland während der COVID-19-Pandemie). *AStA Wirtsch Sozialstat Arch* **16**, 5–20, (2022). link.springer.com/article/10.1007/s11943-021-00297-w

[8] De Nicola G., Kauermann G.: An update on excess mortality in the second year of the COVID-19 pandemic in Germany (Ein Update zur Übersterblichkeit im zweiten Jahr der COVID-19 Pandemie in Deutschland). *AStA Wirtsch Sozialstat Arch* **16**, 21–24 (2022). doi.org/10.1007/s11943-022-00303-9

[9] Kowall B., Standl F., Oesterling F., Brune B., Brinkmann M., Dudda M., Pflaumer P., Jöckel K., Stang A.: Excess mortality due to Covid-19? A comparison of total mortality in 2020 with total mortality in 2016 to 2019 in Germany, Sweden and Spain. *PLoS ONE* **16**(8): e0255540 (2021). doi.org/10.1371/journal.pone.0255540

[10] Levitt M., Zonta F., Ioannidis J.P.A.: Comparison of pandemic excess mortality in 2020–2021 across different empirical calculations. *Environ Res.* **213**: 113754 (2022) doi.org/10.1016/j.envres.2022.113754

[11] World Health Organization: Global Excess Deaths Associated with COVID-19, January 2020–December 2021. https://www.who.int/data/stories/global-excess-deaths-associated-with-covid-19-january-2020-december-2021 (Accessed on May 6, 2022)

[12] Federal Statistical Office of Germany: Number of deaths and excess mortality (Sterbefallzahlen und Übersterblichkeit). www.destatis.de/DE/Themen/Querschnitt/Corona/Gesellschaft/bevoelkerung-sterbefaelle.html (Accessed on January 19, 2023)

[13] Mølbak K., Mazick A.: European monitoring of excess mortality for public health action (EuroMOMO). *European Journal of Public Health* **23** suppl. 1, ckt126.113 (2013). doi.org/10.1093/eurpub/ckt126.113

[14] Keiding N., Clayton D.: Standardization and control for confounding in observational studies: a historical perspective. *Stat. Sci.* **29**, 529–558 (2014). doi.org/10.1214/13-STS453

[15] Staub K., Panczak R., Matthes K.L., et al.: Historically High Excess Mortality During the COVID-19 Pandemic in Switzerland, Sweden, and Spain. *Annals of Internal Medicine* **175** (4), 523–532 (2022). www.acpjournals.org/doi/10.7326/M21-3824

[16] Federal Statistical Office of Germany: Life tables 2015/17, 2016/18, 2017/2019. www-genesis.destatis.de/genesis//online?operation=table&code=12411-0006 (Accessed on January 19, 2023)

[17] German Association of Actuaries (DAV): Life table DAV 2004R. www.aktuar.de

[18] Federal Statistical Office of Germany: Population statistics. www-genesis.destatis.de/genesis//online?operation=table&code=12411-0006 (Accessed on January 19, 2023)

[19] Federal Statistical Office of Germany: Death statistics. www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Bevoelkerung/Sterbefaelle-Lebenserwartung/Tabellen/sonderauswertung-sterbefaelle.html (Accessed on January 19, 2023)

[20] Federal Statistical Office of Germany: Number of births. www-genesis.destatis.de/genesis//online?operation=table&code=12612-0002 (Accessed on January 19, 2023)

[21] Federal Statistical Office of Germany: Number of stillbirths. Available upon request from the FSOG. (Accessed on January 24, 2023)

[22] Robert Koch Institut: COVID-19 Todesfälle nach Sterbedatum. www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Projekte_RKI/ COVID-19_Todesfaelle.html (retrieved on January 19, 2023)

[23] Robert Koch Institut: Digitales Impfquoten-Monitoring COVID-19. www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Daten/ Impfquotenmonitoring.xlsx?__blob=publicationFile (retrieved on January 19, 2023)

[24] Schwab, C., Domke, L.M., Hartmann, L. et al.: Autopsy-based histopathological characterization of myocarditis after anti-SARS-CoV-2-vaccination. *Clin. Res. Cardiol.* (2022). doi.org/10.1007/s00392-022-02129-5

[25] Paul-Ehrlich-Institute: Bericht über Verdachtsfälle von Nebenwirkungen und Impfkomplikationen nach Impfung zum Schutz vor COVID-19. https://www.pei.de/SharedDocs/Downloads/DE/newsroom/dossiers/sicherheitsberichte/sicherheitsbericht-27-12-bis-31-07-21.pdf (retrieved on January 29, 2023)

Christof Kuhbandner
Universität Regensburg
Institut für Experimentelle Psychologie
93040 Regensburg
Germany

Matthias Reitzner
Universität Osnabrück
Institut für Mathematik
49069 Osnabrück
Germany

8 Supplementary Material

8.1 Yearly Mortality Excess

In Section 2.2, we have stated the total expected number of deaths $\mathbb{E}D_t$ in 2020–2022 only for certain age groups; in the following table, we list the detailed expected number of deaths $\mathbb{E}D_{x,t}$ for males and $\mathbb{E}D_{y,t}$ for females and for each age x, y separately.

age	$\mathbb{E}D_{x,2020}$	$\mathbb{E}D_{y,2020}$	$\mathbb{E}D_{x,2021}$	$\mathbb{E}D_{y,2021}$	$\mathbb{E}D_{x,2022}$	$\mathbb{E}D_{y,2022}$
0	1051	837	1053	838	1051	837
1	411	330	403	322	408	327
2	70	56	68	54	67	53
3	50	41	49	40	49	39
4	43	37	43	37	42	37
5	38	34	39	34	39	35
6	35	29	35	29	36	30
7	32	24	32	24	33	24
8	29	21	29	21	30	21
9	29	19	29	19	29	19
10	29	22	28	21	28	21
11	29	26	28	26	28	26
12	31	29	31	28	30	28
13	37	31	37	31	37	30
14	49	35	48	35	49	35
15	65	43	64	42	64	42
16	85	49	83	48	82	47
17	112	53	110	52	108	51
18	146	62	140	60	138	59
19	171	68	162	65	158	64
20	185	72	175	69	168	66
21	191	73	185	70	178	69
22	198	73	190	70	186	69
23	207	76	202	74	197	71
24	215	80	216	80	214	79
25	218	82	217	82	221	83
26	221	86	215	84	217	84
27	232	98	226	95	222	93
28	251	113	243	109	239	107
29	281	136	263	127	257	124

age	$\mathbb{E}D_{x,2020}$	$\mathbb{E}D_{y,2020}$	$\mathbb{E}D_{x,2021}$	$\mathbb{E}D_{y,2021}$	$\mathbb{E}D_{x,2022}$	$\mathbb{E}D_{y,2022}$
30	313	159	301	153	283	143
31	342	173	338	170	326	164
32	373	181	370	180	368	178
33	396	191	401	192	400	192
34	400	208	408	210	416	213
35	415	224	420	226	430	230
36	463	240	461	238	468	241
37	516	264	507	258	508	257
38	553	298	547	294	540	288
39	589	327	587	324	583	319
40	634	338	643	343	643	341
41	677	354	691	363	704	369
42	714	389	715	391	733	401
43	761	424	765	426	768	429
44	825	457	835	464	842	468
45	911	512	907	510	921	519
46	1020	577	1002	565	1000	566
47	1185	667	1119	633	1101	622
48	1432	815	1296	742	1226	706
49	1686	969	1565	902	1419	824
50	1981	1136	1851	1068	1722	997
51	2346	1334	2188	1250	2048	1178
52	2725	1519	2602	1452	2431	1363
53	3142	1719	3026	1657	2896	1587
54	3572	1932	3477	1887	3355	1823
55	4027	2174	3934	2126	3837	2080
56	4500	2426	4432	2385	4338	2336
57	4867	2614	4917	2631	4851	2592
58	5216	2799	5287	2821	5353	2847
59	5601	3009	5678	3039	5768	3070
60	5967	3229	6074	3287	6169	3327
61	6275	3395	6460	3496	6588	3566
62	6605	3551	6780	3639	6991	3756
63	6991	3784	7081	3818	7280	3921
64	7305	4009	7438	4053	7541	4097
65	7615	4256	7734	4271	7880	4325
66	7923	4519	8039	4539	8168	4564

age	$\mathbb{E}D_{x,2020}$	$\mathbb{E}D_{y,2020}$	$\mathbb{E}D_{x,2021}$	$\mathbb{E}D_{y,2021}$	$\mathbb{E}D_{x,2022}$	$\mathbb{E}D_{y,2022}$
67	8321	4819	8324	4810	8449	4839
68	8767	5190	8694	5159	8700	5154
69	9254	5657	9147	5614	9079	5590
70	9731	6065	9673	6072	9571	6038
71	9780	6144	10155	6440	10098	6458
72	9640	6194	10183	6552	10582	6879
73	9276	6133	10055	6621	10635	7015
74	8621	5867	9697	6532	10528	7064
75	10089	7013	8972	6208	10108	6923
76	12388	8682	10482	7408	9333	6560
77	13162	9438	12930	9297	10965	7951
78	15407	11605	13810	10284	13590	10141
79	18457	14696	16206	12756	14551	11313
80	20017	16839	19386	16175	17062	14053
81	20467	18190	21048	18632	20434	17918
82	20225	19066	21459	20099	22110	20602
83	20114	20102	21052	20796	22377	21961
84	20104	21285	20790	21697	21806	22506
85	19414	21854	20524	22790	21299	23292
86	16879	20329	19502	23152	20686	24195
87	14702	19213	16687	21203	19333	24195
88	14305	20118	14286	19596	16274	21718
89	13939	21297	13524	20135	13539	19632
90	12811	21495	12710	20897	12381	19758
91	11210	20458	11421	20568	11357	20012
92	9394	18809	9797	19013	9982	19172
93	7378	16790	7937	16852	8243	17080
94	5678	14855	6000	14498	6427	14573
95	4077	12447	4461	12477	4696	12182
96	2841	9845	3118	10055	3380	10057
97	2086	7628	2116	7584	2284	7723
98	1509	5910	1502	5676	1495	5643
99	1057	4389	1062	4296	1046	4087
100	1085	4294	1236	4835	1368	5108
≥ 101	646	2497	805	2970	929	3393
total	488,440	493,117	494,269	495,439	500,190	498,355

8.2 Mortality prediction using different life tables

We list the expected number of deaths for 2020, 2021, and 2022 for certain age groups using the life tables 2015/17, 2016/18, 2017/19 of the Federal Statistical Office of Germany and half the DAV2004R-longevity factors.

age range $\bar{a}$	LT 2015/17 $\mathbb{E}D_{\bar{a},2020}$	LT 2016/18 $\mathbb{E}D_{\bar{a},2020}$	LT 2017/19 $\mathbb{E}D_{\bar{a},2020}$	observed $d_{\bar{a},2020}$
0-14	3,565	3,585	3,531	3,306
15-29	4,070	3,996	3,944	3,844
30-39	6,718	6,655	6,626	6,668
40-49	15,673	15,557	15,345	15,507
50-59	60,494	59,796	58,641	57,331
60-69	116,457	117,236	117,432	118,460
70-79	197,146	197,428	198,389	201,957
80-89	387,256	382,712	378,459	378,406
90-∞	198,585	199,056	199,191	200,093
0-∞	989,964	986,021	981,557	985,572
	$\mathbb{E}D_{\bar{a},2021}$	$\mathbb{E}D_{\bar{a},2021}$	$\mathbb{E}D_{\bar{a},2021}$	$d_{\bar{a},2021}$
0-14	3,538	3,559	3,513	3,368
15-29	3,938	3,866	3,817	3,934
30-39	6,677	6,614	6,585	6,812
40-49	15,190	15,081	14,877	16,095
50-59	59,526	58,844	57,705	59,350
60-69	117,481	118,264	118,456	126,781
70-79	188,917	189,319	190,335	204,839
80-89	401,711	396,993	392,535	398,041
90-∞	201,236	201,753	201,884	204,467
0-∞	998,213	994,294	989,707	1,023,687
	$\mathbb{E}D_{\bar{a},2022}$	$\mathbb{E}D_{\bar{a},2022}$	$\mathbb{E}D_{\bar{a},2022}$	$d_{\bar{a},2022}$
0-14	3,542	3,563	3,517	3,527
15-29	3,872	3,802	3,755	4,115
30-39	6,637	6,574	6,546	7,130
40-49	14,901	14,798	14,601	15,653
50-59	58,244	57,583	56,471	56,554
60-69	119,031	119,820	119,983	128,370
70-79	184,604	185,135	186,303	205,435
80-89	414,107	409,349	404,994	421,201
90-∞	201,682	202,246	202,375	219,191
0-∞	1,006,620	1,002,869	998,545	1,061,176

8.3 The empirical standard deviation for different age groups

In the following table, we list the results of the linear regression model for the age groups we used in this paper. Using the values $d_{\bar{a},t}$ of the years $t = 2010, \ldots, 2019$ from the Federal Statistical Office of Germany, we compute the parameters $\beta_{0,\bar{a}}, \beta_{\bar{a}}$ of the regression function

$$d_{\bar{a},t} \approx L_{\bar{a}}(t) = \beta_{0,\bar{a}} + \beta_{\bar{a}} \cdot t.$$

The empirical variance $\hat{\sigma}(d_{\bar{a},t})^2$ is given by the estimate

$$\frac{1}{9} \sum_{t=2010}^{2019} \Big(d_{\bar{a},t} - L_{\bar{a}}(t) \Big)^2$$

and the square root yields the empirical standard deviation. We list for each age group of interest the parameters $\beta_{0,\bar{a}}, \beta_{\bar{a}}$ and $\hat{\sigma}(d_{\bar{a},t})$.

age range $\bar{a}$	$\beta_{0,\bar{a}}$	$\beta_{\bar{a}}$	$\hat{\sigma}(d_{a,t})$
0-14	-48,080	26	158
15-29	236,672	-115	148
30-39	-72,304	39	245
40-49	1,998,315	-982	237
50-59	-235,571	146	868
60-69	-3,528,834	1,804	3,646
70-79	2,780,547	-1,272	6,101
80-89	-8,817,920	4,539	7,770
90-∞	-14,249,538	7,152	4,005
30-49	1,926,011	-943	427
60-79	-748,287	532	5,088
80-∞	-23,067,459	11,692	9,924
total	-21,936,714	11,336	14,162

8.4 Monthly expected mortality: allocation factors

We list the estimated proportion of deaths in month m and different age ranges $\bar{x}, \bar{y}$. The first table lists the results $f_{\bar{x},m}$ for the male population, the second $f_{\bar{y},m}$ for the female population, both in percentage.

$\bar{x} \diagdown m$	1	2	3	4	5	6	7	8	9	10	11	12
0-15	8.8	8.1	9.1	8.3	8.1	8.5	8.5	8.5	7.9	8.1	7.8	8.4
15-30	8.5	7.6	8.3	8.3	8.7	8.6	9.2	8.7	8.2	8.3	8.0	7.7
30-35	8.7	7.7	8.6	8.5	8.6	8.4	8.8	8.5	7.7	8.4	8.0	8.1
35-40	8.3	7.9	8.8	8.2	8.7	8.1	8.7	8.6	7.8	8.3	7.9	8.8
40-45	8.9	8.3	9.1	8.4	8.4	8.0	8.4	8.3	8.1	8.1	8.0	8.1
45-50	9.2	8.2	8.9	8.3	8.3	8.2	8.4	8.2	7.9	8.2	7.9	8.2
50-55	9.0	8.2	9.0	8.2	8.4	8.1	8.2	8.2	7.8	8.3	8.1	8.3
55-60	8.9	8.3	9.1	8.3	8.4	8.0	8.3	8.0	7.8	8.3	8.2	8.4
60-65	8.9	8.3	8.9	8.2	8.2	8.0	8.3	8.1	7.7	8.4	8.3	8.8
65-70	8.9	8.5	9.1	8.2	8.2	7.8	8.2	8.1	7.7	8.3	8.1	8.8
70-75	9.1	8.7	9.3	8.3	8.3	7.8	8.2	7.9	7.6	8.2	8.1	8.6
75-80	9.1	8.6	9.4	8.3	8.2	7.7	8.0	7.8	7.6	8.2	8.2	8.9
80-85	9.1	8.7	9.3	8.3	8.1	7.6	8.0	7.8	7.5	8.3	8.3	9.1
85-90	9.2	8.8	9.4	8.3	8.0	7.5	7.9	7.8	7.4	8.3	8.3	9.2
90-95	9.1	8.9	9.4	8.1	7.9	7.4	7.7	7.6	7.4	8.5	8.5	9.5
95-∞	9.7	9.1	9.8	8.3	7.8	7.3	7.6	7.3	7.3	8.3	8.2	9.2

$\bar{y} \diagdown m$	1	2	3	4	5	6	7	8	9	10	11	12
0-15	8.8	8.5	9.1	8.1	8.0	8.5	8.0	7.9	8.1	8.2	7.9	8.9
15-30	8.7	8.6	8.8	8.1	8.2	7.9	8.5	8.4	8.3	8.0	7.9	8.6
30-35	8.5	7.6	9.1	8.5	8.1	8.3	7.9	8.7	8.2	8.3	7.9	9.2
35-40	8.4	8.0	8.5	8.2	8.6	8.1	8.4	8.6	8.2	8.2	8.0	8.8
40-45	9.1	8.5	9.2	8.2	8.3	8.1	8.1	7.9	7.9	8.3	8.0	8.2
45-50	9.0	8.3	9.0	8.2	8.3	8.2	8.2	8.1	7.8	8.4	8.1	8.4
50-55	8.8	8.3	8.9	8.1	8.3	7.9	8.2	8.1	8.0	8.3	8.5	8.6
55-60	8.9	8.3	8.8	8.2	8.3	7.9	8.2	8.1	7.9	8.4	8.3	8.7
60-65	8.9	8.4	9.1	8.1	8.3	8.0	8.2	8.1	7.7	8.2	8.2	8.8
65-70	9.0	8.5	9.2	8.2	8.2	7.8	8.2	8.0	7.7	8.3	8.1	8.7
70-75	9.1	8.7	9.3	8.4	8.2	7.7	8.1	7.9	7.7	8.2	8.0	8.7
75-80	9.1	8.6	9.4	8.3	8.1	7.7	8.0	8.0	7.6	8.2	8.2	8.8
80-85	9.1	8.9	9.5	8.3	8.0	7.6	8.0	7.9	7.5	8.1	8.2	8.9
85-90	9.4	9.1	9.8	8.4	8.0	7.4	7.9	7.7	7.4	8.0	8.0	8.8
90-95	9.2	9.0	9.7	8.3	7.9	7.4	7.9	7.7	7.4	8.1	8.2	9.2
95-∞	9.6	9.3	10.0	8.3	7.8	7.2	7.8	7.6	7.2	8.1	8.2	9.0

In addition, we state the empirical standard deviation of $\frac{d_{\bar{x},t,m}}{d_{\bar{x},t}}$ and $\frac{d_{\bar{y},t,m}}{d_{\bar{y},t}}$ around their mean values $f_{\bar{x}}m$ and $f_{\bar{y},m}$, again in percentage.

$\bar{x} \diagdown m$	1	2	3	4	5	6	7	8	9	10	11	12
0-15	0.8	0.8	1.0	0.5	0.6	0.5	1.0	0.6	0.6	0.5	0.6	1.0
15-30	0.7	0.6	0.8	0.7	0.8	0.5	0.6	0.6	1.1	0.9	0.4	0.9
30-35	0.8	0.7	0.5	0.7	0.6	0.4	0.6	0.5	0.3	0.5	0.5	0.9
35-40	0.7	0.6	0.5	0.5	0.5	0.6	0.5	0.6	0.6	0.5	0.6	0.8
40-45	0.5	0.6	0.4	0.4	0.3	0.4	0.3	0.3	0.4	0.2	0.4	0.6
45-50	0.3	0.3	0.5	0.2	0.3	0.3	0.4	0.2	0.2	0.4	0.4	0.4
50-55	0.4	0.3	0.5	0.3	0.2	0.3	0.2	0.3	0.2	0.2	0.2	0.3
55-60	0.3	0.5	0.5	0.3	0.3	0.2	0.2	0.2	0.3	0.2	0.4	0.6
60-65	0.3	0.3	0.5	0.2	0.2	0.3	0.2	0.2	0.2	0.2	0.3	0.3
65-70	0.5	0.6	0.7	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.2	0.5
70-75	0.4	0.5	0.6	0.2	0.2	0.2	0.3	0.3	0.2	0.3	0.3	0.5
75-80	0.5	0.6	0.7	0.3	0.2	0.2	0.3	0.2	0.2	0.2	0.3	0.4
80-85	0.6	0.9	0.8	0.3	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.5
85-90	0.9	1.1	1.0	0.5	0.4	0.2	0.4	0.4	0.3	0.4	0.5	0.7
90-95	0.9	1.1	1.0	0.7	0.5	0.2	0.4	0.6	0.4	0.4	0.7	0.9
95-∞	0.8	0.8	0.9	0.2	0.4	0.4	0.5	0.7	0.4	0.2	0.2	0.5

$\bar{y} \diagdown m$	1	2	3	4	5	6	7	8	9	10	11	12
0-15	0.8	1.0	1.1	0.9	0.8	1.1	0.9	0.9	1.0	0.8	1.0	0.8
15-30	0.9	1.5	0.9	1.1	0.9	1.0	0.9	0.8	1.0	1.1	0.4	1.3
30-35	0.8	1.0	1.0	0.6	1.4	0.8	0.9	0.7	0.7	0.6	1.0	0.9
35-40	1.1	0.8	1.1	0.7	0.4	0.5	0.5	0.8	0.7	0.8	0.5	0.7
40-45	0.5	0.4	0.7	0.3	0.3	0.4	0.5	0.5	0.5	0.3	0.4	0.6
45-50	0.3	0.4	0.4	0.2	0.3	0.4	0.4	0.3	0.4	0.3	0.6	0.4
50-55	0.3	0.3	0.7	0.3	0.3	0.3	0.3	0.2	0.3	0.3	0.4	0.3
55-60	0.3	0.4	0.8	0.3	0.2	0.3	0.3	0.2	0.4	0.3	0.3	0.5
60-65	0.3	0.5	0.6	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.4	0.3
65-70	0.5	0.6	0.7	0.3	0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.4
70-75	0.5	0.6	0.7	0.2	0.2	0.2	0.3	0.2	0.2	0.3	0.3	0.4
75-80	0.5	0.7	0.8	0.3	0.2	0.2	0.3	0.3	0.2	0.2	0.3	0.4
80-85	0.6	0.9	1.0	0.4	0.2	0.3	0.4	0.2	0.3	0.3	0.4	0.5
85-90	0.9	1.2	1.1	0.4	0.2	0.3	0.4	0.3	0.4	0.4	0.6	0.8
90-95	0.9	1.1	1.2	0.6	0.2	0.3	0.5	0.3	0.4	0.4	0.6	0.8
95-∞	0.7	0.8	1.0	0.3	0.3	0.4	0.4	0.3	0.4	0.2	0.3	0.4

8.5 Monthly development: age group 0-14

We list the total expected monthly number of deaths $\mathbb{E}D_{\bar{a},t,m}$ for children, $\bar{a} = [0, 14]$, the observed number of deaths, and the relative difference.

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	309 272	-11.96%	308 273	-11.45%	309 271	-12.19%
m=2	302 291	-3.52%	291 215	-26.02%	291 271	-6.85%
m=3	319 313	-2.01%	319 277	-13.10%	319 280	-12.26%
m=4	288 289	0.24%	288 255	-11.37%	288 264	-8.34%
m=5	285 277	-2.72%	284 305	7.34%	284 276	-2.97%
m=6	299 275	-7.97%	298 292	-2.08%	299 327	9.54%
m=7	292 278	-4.82%	291 283	-2.91%	292 311	6.59%
m=8	290 273	-5.98%	290 297	2.49%	290 303	4.45%
m=9	281 277	-1.55%	281 287	2.21%	281 298	6.02%
m=10	286 260	-8.99%	285 300	5.23%	285 282	-1.19%
m=11	276 240	-13.02%	275 302	9.68%	276 295	7.02%
m=12	304 261	-14.10%	303 282	-6.99%	304 349	14.98%

8.6 Monthly development: the exceptional age group 50-59

We list the total expected monthly number of deaths $\mathbb{E}D_{\bar{a},t,m}$ for the age group $\bar{a} = [50, 59]$, the observed number of deaths, and the relative difference.

	$t = 2020$		$t = 2021$		$t = 2022$	
	expected observed	rel.diff.	expected observed	rel.diff.	expected observed	rel.diff.
m=1	5,215 5,102	-2.18%	5,147 5,438	5.65%	5,037 5,013	-0.47%
m=2	5,011 4,699	-6.23%	4,776 4,600	-3.68%	4,674 4,492	-3.89%
m=3	5,242 5,036	-3.94%	5,174 4,853	-6.20%	5,063 4,811	-4.98%
m=4	4,822 4,743	-1.65%	4,760 5,166	8.53%	4,659 4,688	0.63%
m=5	4,894 4,742	-3.10%	4,830 5,014	3.82%	4,726 4,552	-3.69%
m=6	4,666 4,547	-2.56%	4,605 4,736	2.85%	4,505 4,482	-0.52%
m=7	4,810 4,647	-3.39%	4,748 4,686	-1.30%	4,647 4,759	2.42%
m=8	4,743 4,629	-2.40%	4,680 4,631	-1.05%	4,579 4,641	1.35%
m=9	4,599 4,461	-2.99%	4,538 4,582	0.96%	4,441 4,394	-1.06%
m=10	4,873 4,755	-2.42%	4,809 4,962	3.17%	4,707 4,797	1.92%
m=11	4,798 4,753	-0.94%	4,735 5,094	7.57%	4,634 4,561	-1.58%
m=12	4,967 5,217	5.03%	4,903 5,588	13.97%	4,799 5,364	11.77%

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