Physica A 465 (2017) 425–437

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Money creation and circulation in a credit economy

Wanting Xiong, Han Fu, Yougui Wang*

School of Systems Science, Beijing Normal University, Beijing 100875, People's Republic of China

HIGHLIGHTS

- An agent-based model is developed to study money creation and circulation.
- The money multiplier is determined by not only borrowing but also repayment.
- The velocity of money depends on both money-related and debt-related factors.

ARTICLE INFO

Article history:
Received 20 December 2014
Received in revised form 31 October 2015
Available online 26 August 2016

Keywords:
Money creation
Money multiplier
Velocity of money circulation
Debt
Econophysics

ABSTRACT

This paper presents a multi-agent model describing the main mechanisms of money creation and money circulation in a credit economy. Our special attention is paid to the role of debt in the two processes. With the agent-based modeling approach, macro phenomena are well founded in micro-based causalities. A hypothetical economy composed of a banking system and multiple traders is proposed. Instead of being a pure financial intermediary, the banking system is viewed as the center of money creation and an accelerator of money circulation. Agents finance their expenditures not only by their own savings but also through bank loans. Through mathematical calculations and numerical simulation, we identify the determinants of money multiplier and those of velocity of money. In contrast to the traditional money creation model, the money multiplier is determined not only by the behavior of borrowing but also by the behavior of repayment as well. The velocity of money is found to be influenced by both money-related factors such as the expenditure habits of agents with respect to their income and wealth and debt-related factors such as borrowing and repayment behaviors of debtors and the reserve requirements faced by banks.

1. Introduction

The issue of how debt affects the economy has been mentioned for a long time but was not attractive to mainstream economists until the recent crisis. Except for a few attempts [1–4] to introduce debt into their models, mainstream macroeconomists have a longstanding habit of ignoring debt [5] and have spent tremendous effort in proving the legitimacy of continuing this tradition [6]. In contrast to these theorists, however, empiricists found that debt, instead of being something that could be ignored, is actually the driving force behind the ups and downs in the economy [7–12]. Because of the inability of current macroeconomics to predict and explain the current crisis, there is growing appeal for it to be revolutionized so that it takes the financial sector seriously. It is therefore obvious that debt, which is central to finance, requires a repositioning to match its real significance.

Evidence of the great influence of debt on the economy can be found in all crises, and this time was no exception [7–9]. The Great Depression in the 1930s, for example, was triggered not only by the reduction of the monetary base but also by the

* Corresponding author.
E-mail address: ygwang@bnu.edu.cn (Y. Wang).

http://dx.doi.org/10.1016/j.physa.2016.08.023

contraction of credit [1]. The 2008 financial crisis and the 2009 EU debt crisis, once again, revealed the causal link between financial crises and rapid surges in debt [8]. As a matter of fact, there is a strong correlation between the change in debt and the change in macroeconomic activities in both critical and normal times [13,14].

Regarding all periods, opinions are divided between mainstream economists who attribute all fluctuations to real factors and others who think that monetary factors should receive much greater attention. Paul Krugman, one opinion leader and one of the first economists who acknowledged the failure of macroeconomics [15], is the best representative of current mainstream opinion on money and debt. In his recent work, he insisted that the level of debt is not something we should worry about and financial institutions, instead of being the center of credit creation, are merely intermediaries that could be ignored [16]. These arguments stand in stark contrast to those that place strong emphasis on the role of the financial sector [17–19]. Several big names can be listed. Fisher attributed deflation brought on by the crisis to debt liquidation [20]. Minsky proposed the financial instability hypothesis based upon the distinction of three types of borrowers [21]. All of these theories provide various ways to understanding the impacts of debt on the economy.

To study the influence of debt on the economy, one of the most direct and popular ways is to study its impact on aggregate demand [2,22–25]. Steve Keen, in his early work, argued that aggregate demand is the summation of preceding income and change in debt [18]. Afterwards, he changed the expression into the sum of preceding income and the product of the change in debt and the velocity of money [19]. Regardless of which expression is correct, it can be implied that debt influences both money creation and money circulation.

The act of borrowing and lending is the core of money creation. When a commercial bank lends to a borrower, both agents' balance sheets will expand with money (asset to the borrower and liability to the commercial bank[1]) and debt (asset to the commercial bank and liability to the borrower) being simultaneously created. In the textbook story of money creation, the quantity of loans commercial banks could possibly grant is constrained by the quantity of base money and reserve requirement. Models of this kind, however, are often criticized for being static and deficient in micro foundations. Due to the lack of the concepts of stocks and flows, these models often neglect the time and dynamics of the money creation process. Even if there is time, it is hypothetical logic time rather than realistic historical time. The behaviors that constitute the money creation process are characterized by ratios of stocks such as the ratio of currencies to deposits which considers no time and provides no accounts for how the frequency of borrowing or spending could make a difference on the money aggregate. With everything covered up by ratios of macro variables, important individual behaviors are ignored. One example of this kind is the common omission of the repayment behavior. Economists talk about banks' lending behavior but hardly consider the consequences of loan repayments. Actually, when debtors repay their debt, money already created before will be annihilated [26,27]. Analogous to positrons and negatrons, debt and money are found to arise or disappear simultaneously [28]. It is demonstrated by Siyan Chen et al. [29] through a random exchange model that the repayment behavior makes a difference to the economic equilibrium despite its idealist postulations of random exchange and the simple characterizations of banks.

The creation and destruction of debt also affect the process of money circulation. Since debt is a useful means to get money where it is most needed, from creditors with an excess of it, to borrowers who are short of it, it enables debtors to make consumptions that would have been impossible without the loan. In other words, the creation of debt will bring about more transactions. One way to describe the impacts of debt on the circulation of money is to study its impacts on the velocity of money, which is the frequency at which one unit of money participates in transactions. The velocity of money circulation is a central matter in monetary theories which has attracted much attention for hundreds of years. Although exploration of the velocity can be traced backward to the earlier works in 1660s [30], most of current investigations of velocity is commonly attributed to Irving Fisher's exchange equation, $MV = PY$, where M is the total stock of money, V is the circulation velocity of money, P is the average price level and Y is the total output [31]. From this equation, the velocity of money can be computed as the ratio of transaction volume or aggregate income to money stock. Based on this equation, many theoretical and empirical research works on the velocity have been carried out to examine its determinants [32–41]. However, the Fisher Equation does not look into the intrinsic properties of the velocity. In the microscopic view, the velocity can be measured by the reciprocal of the average holding time of money, which is the time interval between two money transfers [42,43]. Also, it was found that the distribution of wealth and the required reserve ratio [44] would have a critical influence on both the distribution and the velocity of money. However, it remains ambiguous what role debt plays in the process of money circulation.

In this paper, we propose an agent-based model to depict the dynamic process of money creation and money circulation in a credit economy. To simplify the analysis, we employ the textbook assumption of exogenous money where the lending capability of banks is restricted by the money base and the required reserve ratio. Meanwhile, compared to the traditional theories, we make two major extensions. The first extension is to take the repayment behavior into account in the process of money creation. The second is that loans from banks are used to financing expenditures which create money and accelerate money circulation simultaneously. Through computer simulations, we demonstrate macroscopic impacts of individual behaviors and institutional mechanisms, which are specifically embodied in two variables, the money multiplier and the velocity of money. Section 2 presents the existing theories of money creation and money circulation. Section 3 illustrates the model in detail. Theoretical analysis of the model is performed in Section 4, while the simulation results are presented in Section 5. We finish with some conclusions in the last section.

[1] Here we suppose that the loan is given in the form of deposit. If the loan is in the form of currency, then money will be the liability of the central bank.

2. Traditional theories of money creation and money circulation

2.1. Money creation

The story of money creation was first put forward by Brunner and Meltzer [45,46] and prevails in current textbooks. In this model, a simplified banking system can be viewed as the combination of two parts: a central bank that sets monetary policies including the amount of money base and the ratio of required reserves and commercial banks that provide savings and loan services to the public. In this artificial economy, money takes the form of currency, denoted as C, and deposits in the commercial banks, denoted as D. Then, the total amount of money, M, can be written as the sum of these two items, i.e.,

$$M = C + D. \tag{1}$$

The monetary base, M_0, after innumerable rounds of saving and lending, exists in the form of currencies and reserves (R):

$$M_0 = C + R. \tag{2}$$

One significant role that the commercial banks play in the economy is to facilitate money creation. Having absorbed deposits, banks can lend part of them, resulting in more deposit accounts, i.e., an increase in the aggregate monetary supply. The portion of currency that has not been lent is known as reserves, held for cash withdrawals. There are two types of reserves, required reserves (RR), which are regulated strictly by the central bank, and excess reserves (ER), which are held voluntarily by commercial banks. Thus, the reserves can be written as the sum of the required reserves and the excess reserves, i.e.,

$$R = RR + ER. \tag{3}$$

Replacing all of the components with ratios to deposits, we can rewrite Eqs. (1) and (2) as follows:

$$M = (c + 1) \cdot D, \tag{4}$$

$$M_0 = (c + \gamma + e) \cdot D \tag{5}$$

where c is the currency–deposit ratio, γ is the required reserve ratio, and e is the excess reserve ratio.

Combining (4) and (5), the monetary multiplier can be derived as follows:

$$m = \frac{M}{M_0} = \frac{c + 1}{c + \gamma + e}. \tag{6}$$

Ignoring the holdings of cash and excess reserves, Eq. (6) can be simplified as

$$m = \frac{1}{\gamma}. \tag{7}$$

Examining Eq. (6), we will find it correct at any time point during or after the money creation process. This is only a reflection of how broad money and the monetary base are decomposed according to their classification structures. Such a decomposition method is implemented in a quasi-static way and is unable to reflect the dynamic process of how money is created and what types of behavior of economic agents are involved. Thus, we may say that the expression of the multiplier just reflects structural information rather than behavior-based procedures.

Moreover, if we advance a few steps further, we can easily find a logic flaw in this traditional story. For every repeated round of loans, commercial banks, under the restriction of the fractional reserve policy, will provide increasingly less loans and eventually run out of loanable funds. In other words, banks have to stop lending when the amount of money gets to its equilibrium level. Obviously, this contradicts our common sense. But what is the problem? The answer is the neglect of repayment. In the equilibrium state where the amount of money stops changing, it is repayment that completes the recycle of loanable funds and makes the lending behavior sustainable.

2.2. Money circulation

Regarding money circulation, one of the most important equations is the quantity equation of money,

$$MV = PY, \tag{8}$$

where M is the stock of money, V is the velocity of money, P is the price level, and Y is the real income. This equation was first articulated by John Stuart Mill [47], and then formulated by Irving Fisher [31]. The two sides of the equation respectively represent the nominal value of aggregate transactions from a monetary perspective and a product perspective. As an aggregate measure of the whole economy, the velocity of money is calculated as the ratio of nominal GDP to the stock of money aggregate:

$$V = \frac{PY}{M}. \tag{9}$$

This expression provides a measurement of the velocity of money but gives little information about the mechanism of how the velocity of money is determined. Just as Rothbard [48] reminds us, ''it is absurd to dignify any quantity with a place in an equation unless it can be defined independently of the other terms in the equation''. Thus, the velocity must have a ''life of its own''. Employing a life-cycle model of consumption, it is found that in an economy without credit activities, the velocity of money is determined by the difference between the expected length of life and that of working periods, which is independent of money stock [49]. However, in a credit economy where the activities of lending, borrowing and repayment are considered, it would be imprudent to assume an independent relationship between the velocity of money and the stock of money. In particular, when an individual is able to finance his spendings not only by his own savings or income but also by bank loans, the actual expenditure may well increase, which forms a larger money flow. In this process, both the velocity of money and the stock of money are affected. As discussed above, once bank lending is conducted, additional money is created. Also by enabling transactions that could not have happened due to the lack of financing, the frequency of transactions will increase which may well result in a greater velocity of money. Taking both factors into consideration, we then ask how we can distinguish the roles of debt in the process of money circulation and that of money creation. In order to answer this question, we investigate the underlying mechanism of both processes by taking the approach of modeling. It is argued that this approach is able to uncover the behavioral foundations of the velocity of money and allows for the discussion of debt naturally [50].

3. The model

We consider a closed economy that is composed of N agents, one central bank and one commercial bank. The central bank is in charge of determining the amount of money base and the required reserve ratio. Agents are endowed with a certain amount of money at the very beginning of the model and then trade with each other randomly. The commercial bank provides depositing services to savers and lending services to borrowers. The economy is considered to be in an insufficient demand situation where extra supply of goods can be realized as long as there is extra demand.

To illustrate how the economy mentioned above runs, we present a flowchart in Fig. 1, in which the logical links between any two steps of the whole procedure are presented. As shown in Fig. 1, time is discrete, there are six steps in each period: (I) planned expenditure, (II) demand for loans, (III) credit creation, (IV) real expenditure, (V) income allocation, and (VI) debt repayment. In the beginning of each time period, each agent decides his expenditure plan for this period. This planned expenditure can be expressed as a function of his current wealth and expected future income. When his planned expenditure exceeds his current wealth, he will finance this extra spending by applying for bank loans. On the other hand, after receiving all of the applications for loans, the commercial bank will decide how much of the loan applications can be satisfied by comparing the total demand for loans with the total amount of loanable funds it has. After the step of credit creation, all agents make expenditures financed by their own money as well as bank loans in transactions with each other. For the economy is closed, the aggregate expenditure turns out to be the aggregate income in the economy. The income is then allocated to the agents randomly, and each agent has his own share. People can use their income either to accumulate wealth or to repay debt to the bank. At the end of each period, three key economic variables of every agent, including amount of money, amount of debt, and the expected income are updated correspondingly, the economic procedure moves on to the next period. The detailed elaboration of this procedure is given as follows.

I. **Planned expenditure**

This model is a period model where time is discrete. First, suppose a certain amount of base money (M_0) is issued by the central bank and is allocated to all agents randomly. Consequently, each person receives an amount of money (m_i for agent i) as his initial wealth. For the sake of simplicity, no cash is considered and money exists only in the form of deposits.

Supposing his current wealth is ω_i and expected future income is $\hat{y}_i$, agent i's planned expenditure e_i^p at time t is given by

$$e_i^p(t) = \alpha \cdot \omega_i(t) + \beta \cdot \hat{y}_i(t), \tag{10}$$

where α, β are two exogenous parameters that characterize people's expenditure habits. The former is the marginal propensity to spend with respect to wealth and the latter is that with respect to income.

Because wealth exists only in the form of money in our model, $\omega_i(t)$ can be replaced with $m_i(t)$, which represents the amount of money that agent i holds at time t. Thus, his planned expenditure is transformed as follows:

$$e_i^p(t) = \alpha \cdot m_i(t) + \beta \cdot \hat{y}_i(t). \tag{11}$$

II. **Demand for loans**

The amount of money agent i can spend during this period is financed by his own wealth as well as loans he receives from banks. If his current amount of money is sufficient to meet the need of planned expenditure, i.e., $m_i(t) \geq e_i^p(t)$, he will spend the exact amount of money equal to his planned expenditure without asking for loans. If his planned spending is beyond his capacity, i.e., $m_i(t) < e_i^p(t)$, he will have to seek credit from the commercial bank. His planned loan (b_i^p) is defined as the difference between his planned expenditure and current amount of money:

$$b_i^p(t) = e_i^p(t) - m_i(t). \tag{12}$$

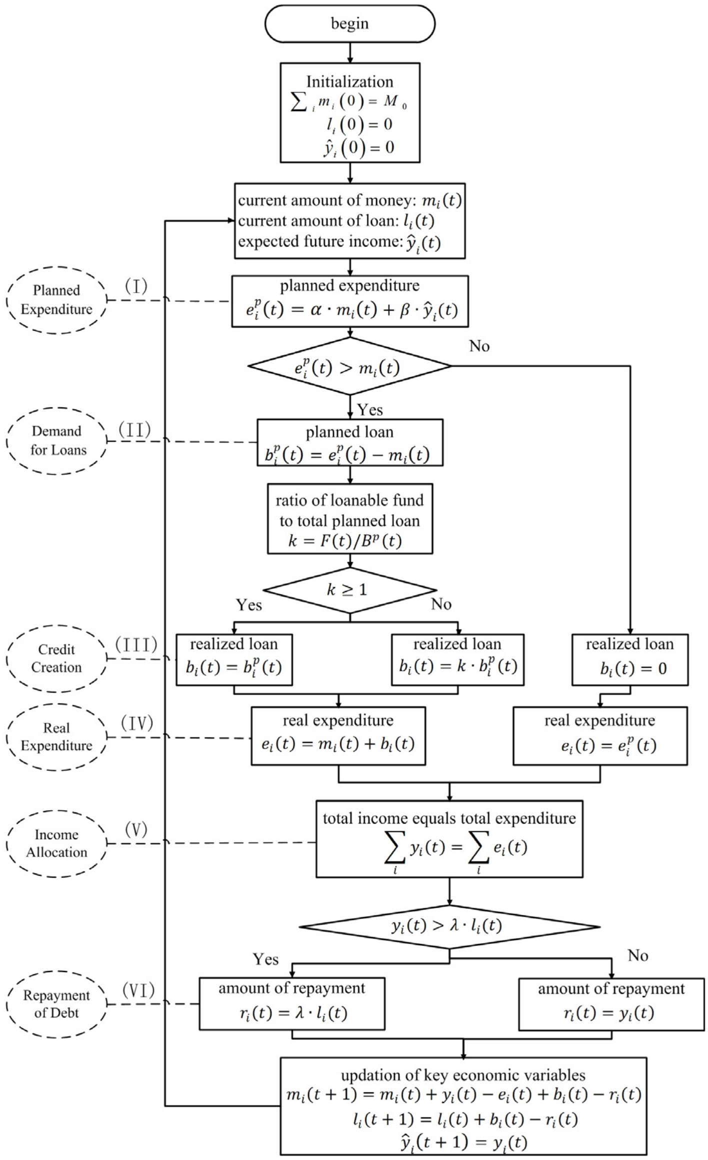

Fig. 1. A flowchart of the economic procedure.

After everyone has submitted his loan application, the commercial bank is confronted with an aggregate demand for loans, which is given by

$$B^p(t) = \sum_i b_i^p(t). \tag{13}$$

III. **Credit creation**

Facing the total amount of planned loans, the banking system decides how much of it could be realized. Regulated by the fractional reserve policy, the loanable funds (F), or the maximum amount of loans that the bank could offer at the current period, should not be more than the excess reserves, which is simply assumed to take the following form:

$$F(t) = (1 - \gamma) \cdot M(t) - L(t), \tag{14}$$

where γ is the required reserve ratio, $M(t)$ is the money aggregate, and $L(t)$ is the outstanding loans at time t. If loanable funds are sufficient to satisfy the aggregate demand for loans, i.e., $F(t) \geq B^p(t)$, all loan applications will be satisfied. Otherwise, if $F(t) < B^p(t)$, the bank could only satisfy a proportion of the loan application. We assume here that the bank grants loans to people in proportion to their individual needs, that is

$$k = F(t)/B^p(t), \tag{15}$$

$$b_i(t) = k \cdot b_i^p(t), \tag{16}$$

where k is a proportional coefficient, and $b_i(t)$ represents agent i's realized loan at time t.

IV. **Real expenditure**

As mentioned above, if agent i does not need to borrow from bank, his real expenditure is exactly his planned one; otherwise, his real expenditure is equal to the sum of current money and borrowed money, which is given by

$$e_i(t) = m_i(t) + b_i(t). \tag{17}$$

By summing up the real expenditure of all agents, the total expenditure in the economy at time t can be obtained as

$$E(t) = \sum_i e_i(t). \tag{18}$$

V. **Income allocation**

Because one's expenditure is another's income, the aggregate income is exactly equal to the total expenditure. After collecting all expenditures, we then allocate the income to each agent according to the uniform distribution, and we thus have

$$Y(t) = \sum_i y_i(t). \tag{19}$$

Here, $y_i(t)$ is the income of agent i at the current period.

VI. **Repayment of debt**

After receiving the income, agents are supposed to repay their debts. They are obliged to repay a fixed proportion of the debt already contracted. The amount of repayment of agent i (r_i) is defined as

$$r_i(t) = Min\{y_i(t), \lambda \cdot l_i(t)\}, \tag{20}$$

where $\lambda \in [0, 1]$ is the proportion of matured debt that should be repaid, and $l_i(t)$ is the stock of debt previously accumulated by agent i. When his income exceeds his obligation of repayment in this period, he pays off all of the required repayment; otherwise, he is forced to return all of the income as debt repayment at time t.

To close the model, we need the following dynamic equations to update the economic variables at time $t+1$. For agent i, his wealth is increased by obtaining income and loans and is decreased by paying expenditure and repayments, that is,

$$m_i(t+1) = m_i(t) + y_i(t) - e_i(t) + b_i(t) - r_i(t). \tag{21}$$

Similarly, loans add to debt, and repayments reduce debt, so the aggregate debt of agent i is given by

$$l_i(t+1) = l_i(t) + b_i(t) - r_i(t). \tag{22}$$

Finally, we can obtain the total amount of money and debt by summing up those of all individuals:

$$M(t+1) = \sum_i m_i(t+1), \tag{23}$$

$$L(t+1) = \sum_i l_i(t+1). \tag{24}$$

At last step, following the assumption of naïve expectation that the expected income for the next period equals that of the current period, we can express the expected future income $\hat{y}_i(t+1)$ of agent i simply as the current income $y_i(t)$.

4. Theoretical analysis

The economic processes described above include two types of interactions: the interaction between agents and the banking system which constitutes the money creation process and the interaction among agents themselves which constitutes the income–expenditure process. Thanks to those debt-related behaviors, these two processes are not independent from each other, but intertwined with complicated feedback loops. For example, the behavior of lending of the banking system does not only changes the stocks of money and debt through the money creation process, but also enables borrowers to make debt-financed expenditure in addition to those financed by their own income or wealth, which then results in changes in the flows of income for others. On the other hand, the behavior of repayment, which is constrained by the income flow for indebted agents in the income–expenditure dynamics among exchanging agents, also affects the money creation process by annihilating money and debt simultaneously. Therefore, when we study the determinants for money multiplier and those for velocity of money circulation, we cannot isolate the two processes.

In addition, we should be aware that there exist two different types of equilibrium where the artificial economy eventually falls into: the stock equilibrium where the stocks stop changing and the flow equilibrium where the flows stop changing. When we simulate the model under the initialization shown in Fig. 1 ($m_i(0)$ is a random share of the money base and $\sum_i m_i(0) = M_0$, $l_i(0) = \hat{y}_i(0) = 0$, for $i = 1, \ldots, N$) and follow the evolution of the stock variables (the money

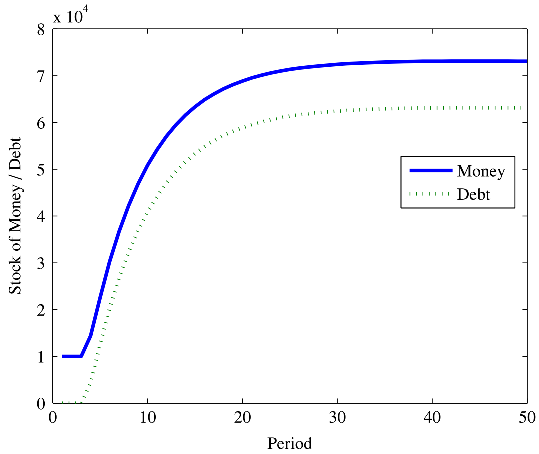

Fig. 2. The evolution of stock of money and the stock of debt as a function of time, where $\alpha = 0.8$, $\beta = 0.8$, $\gamma = 0.05$, $\lambda = 0.1$, $M_0 = 10{,}000$, and $N = 1000$.

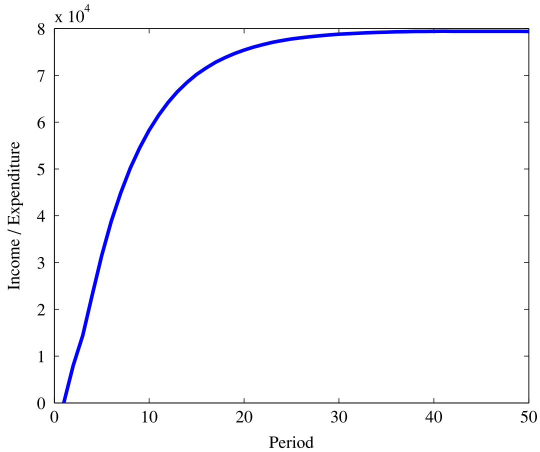

Fig. 3. The evolution of aggregate income/expenditure as a function of time, where $\alpha = 0.8$, $\beta = 0.8$, $\gamma = 0.05$, $\lambda = 0.1$, $M_0 = 10{,}000$, and $N = 1000$.

aggregate and the total amount of debt) and the flow variable (aggregate income/expenditure) as shown in Fig. 2 and Fig. 3 respectively, we can find that the stock equilibrium and the flow equilibrium are not reached at the same time.

With this in mind, we next calculate the equilibrium value for the money multiplier and velocity of money by employing the mean-field approximating approach. Firstly, we consider the change in the stock of debt, which is contributed by the inflow of loans and the outflow of repayments, i.e.,

$$\frac{\mathrm{d}L}{\mathrm{d}t} = B(t) - R(t). \tag{25}$$

Actually, according to the short-side principle, the bank loan B must be not greater than the loanable funds (F). Thus we define a ratio $c_1 \in [0, 1]$ as the portion of the bank loan to the loanable funds, that is to say

$$B = c_1 \cdot F = c_1 \cdot [(1-\gamma)M - L]. \tag{26}$$

It will reach its maximum when all of the loanable funds are lent out.

Likewise, the repayment R is also a portion of the required repayment (λL), which can be expressed as follows by introducing another ratio $c_2 \in [0, 1]$:

$$R = c_2 \cdot \lambda L. \tag{27}$$

Owing to the conservative relation $M = L + M_0$ and M_0 here being constant in time, we can get

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\mathrm{d}L}{\mathrm{d}t}. \tag{28}$$

The equilibrium is defined as the situation where the stocks stop changing, which implies that

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\mathrm{d}L}{\mathrm{d}t} = 0. \tag{29}$$

Solving the above equations yields the equilibrium solution of M:

$$M = \frac{c_1 + c_2\lambda}{c_1\gamma + c_2\lambda} M_0. \tag{30}$$

Consequently, the monetary multiplier m, defined as the ratio of M to M_0, is equal to

$$m = \frac{c_1 + c_2\lambda}{c_1\gamma + c_2\lambda}. \tag{31}$$

This result demonstrates that the money multiplier is determined by the required reserve ratio γ and repayment ratio λ. The higher the required ratio, the larger is the multiplier. The dependence of the multiplier on the repayment ratio seems complex, for it appears in both the denominator and numerator of Eq. (31). Actually, with some simple manipulations, we can see that the multiplier is reversely related to the repayment ratio. That is, the repayment behavior destructs money and naturally reduces the multiplier. We also can draw a conclusion that the multiplier has nothing to do with the parameters of agents' expenditure habits, which implies that no matter how people obtain income and spend their money, the money is always conservative.

If we assume that there is no repayment flow in the economy (i.e., $\lambda = 0$), the money multiplier then degenerates into the following simple form:

$$m = \frac{1}{\gamma}, \tag{32}$$

which is exactly the traditional formulation of the monetary multiplier.

Secondly, we decompose aggregate expenditure E into two parts: one comes from agents' own accounts (A), and the other is the bank loans (B). Therefore, we have

$$Y = E = A + B. \tag{33}$$

Since A should not be more than the planned expenditure E^p, we introduce the third ratio $c_3 \in [0, 1]$ as the ratio of A to E^p so that

$$A = c_3 \cdot E^p = c_3 \cdot (\alpha M + \beta Y). \tag{34}$$

From Eq. (26) we know that the bank loan B is a portion of the loanable funds (F), and it is exactly equal to the repayment (R) at equilibrium state (i.e., $B = R$).

Combining Eqs. (33) (34) and (27), total income Y is just a function of the stock of money M and stock of debts L, that is

$$Y = \frac{c_3\alpha}{1 - c_3\beta} M + \frac{c_2\lambda}{1 - c_3\beta} L. \tag{35}$$

Meanwhile, the relation between the total amount of debts L and the total amount of money M could be given by

$$L = \frac{c_1 - c_1\gamma}{c_1 + c_2\lambda} M. \tag{36}$$

Substituting (36) into (35) and doing a simple manipulation, we then have the expression of velocity of money V as follows,[2]

$$V = \frac{Y}{M} = \frac{c_3\alpha(c_1 + c_2\lambda) + c_2\lambda(c_1 - c_1\gamma)}{(1 - c_3\beta)(c_1 + c_2\lambda)}. \tag{37}$$

From the above set of equations, we can find that income is determined by not only the stocks but also the behavioral parameters shaping the expenditures (α and β) and repayment flow (λ). The equilibrium values of money and debt are correlated with each other and are the outcome of agents' interactions. As a result, the velocity of money is an integration of all of these parameters. Obviously, the velocity of money positively depends on the expenditure parameters (α and β). Furthermore, the repayment ratio λ also has a positive impact on the velocity. Only the required reserve ratio γ contributes to the velocity reversely. The way it takes its effect on the velocity is through influencing the relation between money and debt.

Eqs. (31) and (37) indicate that the equilibrium values of the two key variables related to money creation and circulation can be formulated in terms of parameters that characterize corresponding micro behaviors of agents and the interactions among different sectors.

2 Here we assume that the price level $P = 1$.

5. Simulation results

Based upon the above description of the model, we can deduce that the equilibrium state of the economy is determined by the following four exogenous parameters: the propensity to expend with respect to wealth and income, respectively, α and β, the required reserve ratio γ, and debt repayment ratio λ. To see how the values of these parameters affect the two kinds of equilibrium we mentioned above, we observe the equilibrium money aggregate and total income under different settings. The effects of the four parameters on the monetary multiplier m and the circulation velocity of money V are then presented. We set $N = 1000$ agents involved in all the simulations.

5.1. Proportion parameters

First, we discuss how the equilibrium values of c_1, c_2, and c_3 depend on the values of α and β. The simulations were conducted under the conditions $\gamma = 0.1$ and $\lambda = 0.1$. In Fig. 4, the color depth of each small square is correlated with the value of each proportion parameter, and the color gets darker as the value gets larger.

In Fig. 4(a), the lower-left white area shows that the proportion $c_1 = B/F = 0$, which means that, with α and β in this range, no loan has ever been made (i.e., $B = 0$). As a result, the aggregate debt is zero in this case (i.e., $L = 0$). This area would hence be named *No-Loan Area* for convenient reference. With respect to the proportion $c_2 = R/(\lambda L)$, *No-Loan Area* in Fig. 4(b) is shaded with dots because it is meaningless in this case due to a zero denominator. As shown in Fig. 4(c), the proportion $c_3 = A/E^p$ turns out to be 1 in the *No-Loan Area* since all of the planned expenditure of individuals will be satisfied with what they have already possessed with this setting.

When we turn to the other part of each panel, we can see that the corresponding area is filled with color gradients in Fig. 4(c), which is called the *Loan Area*. The corresponding *Loan Area* in Fig. 4(a) and (b) indicates that the bank lends all it can offer and agents repay the exact amount of money they owe. Fig. 4(c) depicts the situation where planned expenditure cannot be fully realized by individual accounts, and the realized portion gets smaller when α and β get larger.

It is noteworthy that there is a sharp line between the *Loan Area* and the *No Loan Area* in all subfigures in Fig. 4, which indicates a simultaneous phase transition of the three proportion parameters as the values of α and β vary.

5.2. Money multiplier

Fig. 5 demonstrates how the monetary multiplier varies with different values of α and β with $\gamma = 0.1$ and $\lambda = 0.1$. As shown in this figure, the monetary multiplier remains 1 in the *No-Loan Area*, which indicates that no additional money is created through credit creation. In the *Loan Area*, we can see that the money multiplier reaches its maximum value rapidly. The experimental maximum value of the multiplier is 5.4264, which is in the error-allowed range, in accordance with the theoretical value of the money multiplier, 5.5, which is calculated by Eq. (31) with $c_1 = 1, c_2 = 1, \gamma = 0.1$ and $\lambda = 0.1$.

Because γ and λ play no role in the *No-Loan Area*, we examine how m evolves in the space of γ and λ under the settings of $\alpha = 0.8$, $\beta = 0.9$ (a point in *Loan Area*), as shown in Fig. 6. Because the monetary multiplier is 186.5828 when $\lambda = 0$, $\gamma = 0$, which is a massive level compared with other situations, we set a lower bound of 0.05 for both parameters for the sake of a clearer presentation. In general, the deposit reserve ratio does not exceed 0.5, so 0.5 is set to be an upper bound for γ, and 1 for λ.

As we can observe in Fig. 6, the monetary multiplier is not only determined by the required reserve ratio γ, as generally accepted in a conventional sense, but also greatly affected by the speed of debt repayment in the society. When a higher required reserve ratio is adopted by the bank, obtaining a loan becomes harder, and hence, the monetary multiplier is decreased. However, as the repayment proportion becomes higher, more credit money is destructed by the repayment and therefore this will lead to a reduction in the monetary multiplier as well.

To be specific, while keeping λ unchanged as 0.1, the changing relation between the monetary multiplier m and repayment proportion γ is demonstrated in Fig. 7(a), where blue circles refer to simulated data points, and the red line refers to the theoretical prediction with $c_1 = c_2 = 1$, $\lambda = 0.1$. Similarly, the inverse relation between the monetary multiplier m and the repayment proportion λ (for $\gamma = 0.1$) in Fig. 7(b) could be easily understood. It can be observed that the theoretical line fits the experimental data well, with small errors excluded.

5.3. Circulation velocity of money

To explore the impacts of parameter values of α and β on the velocity of money V, Fig. 8 is presented. Two different settings of γ and λ are considered, which are (a) $\gamma = 0.1$, $\lambda = 1$, and (b) $\gamma = 0.1$, $\lambda = 0.1$. It can be observed in this figure that the velocity changes gradually in the *No-Loan Area* and is maximized in the Loan Area under both settings. In the *No-Loan Area*, the realized expenditure is solely determined by the planned expenditures, that is

$$Y = E = \alpha M + \beta Y, \tag{38}$$

from which we can obtain that

$$Y = \frac{\alpha}{1-\beta} M. \tag{39}$$

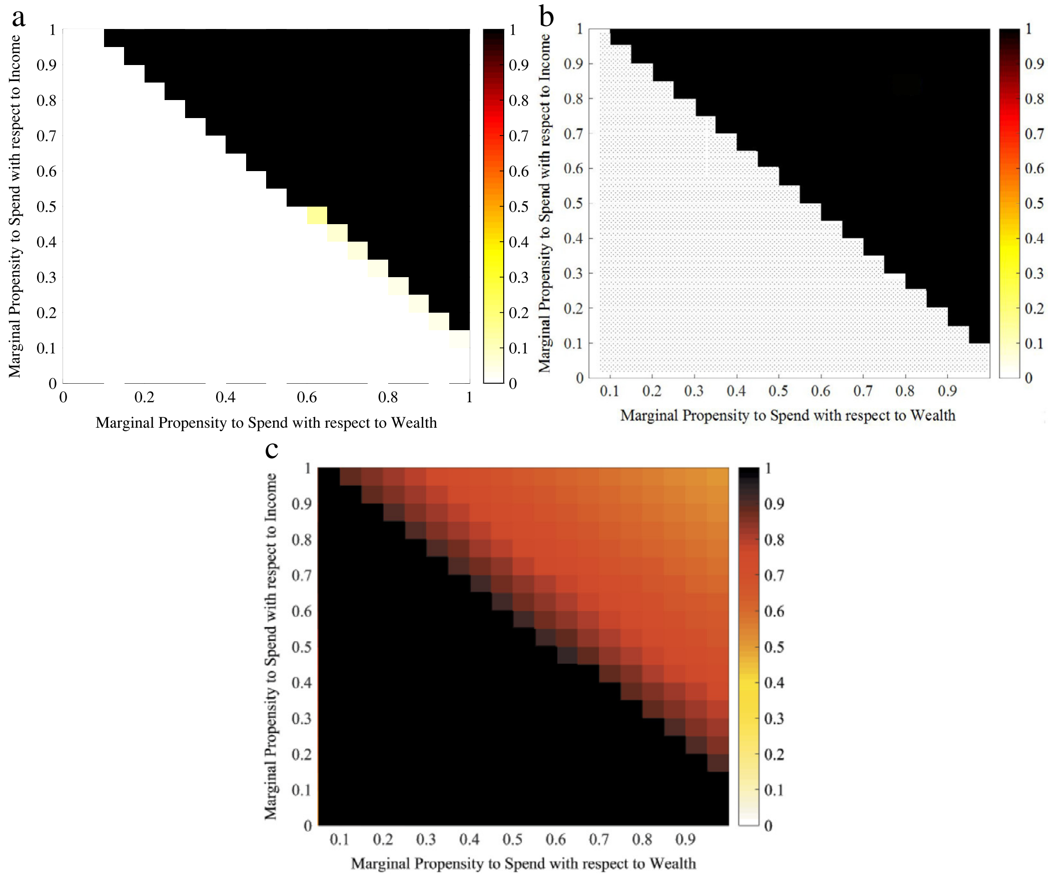

Fig. 4. Dependence of the three proportions $c_1 = B/F(a)$, $c_2 = R/(\lambda L)(b)$, and $c_3 = A/E^p(c)$ on α and β. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

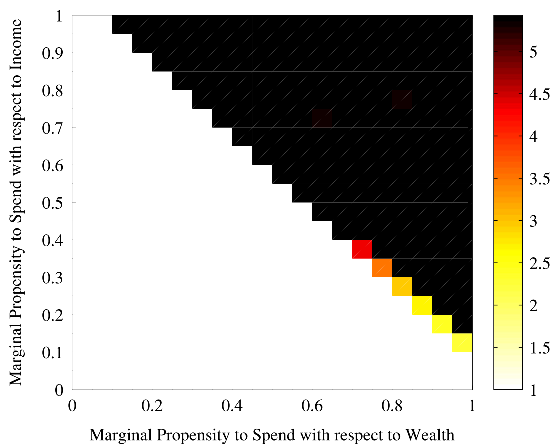

Fig. 5. Monetary multiplier as a function of α and β (for $\gamma = \lambda = 0.1$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Then the expression of V can be deduced as follows:

$$V = \frac{\alpha}{1-\beta}. \tag{40}$$

Such a result confirms the color gradients in the *No-Loan Area* in Fig. 8 and the tendency that velocity increases with increasing α or β.

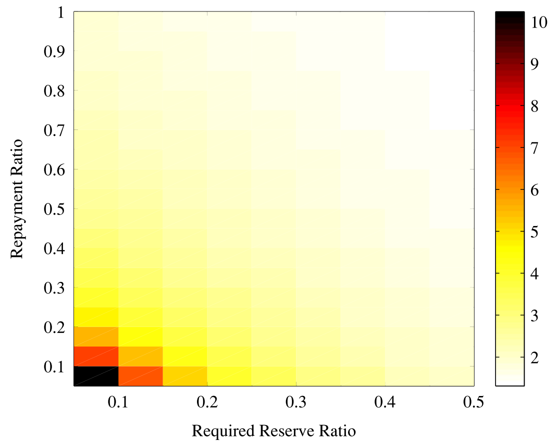

Fig. 6. Monetary multiplier as a function of γ and λ (for $\alpha = 0.8$, $\beta = 0.9$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

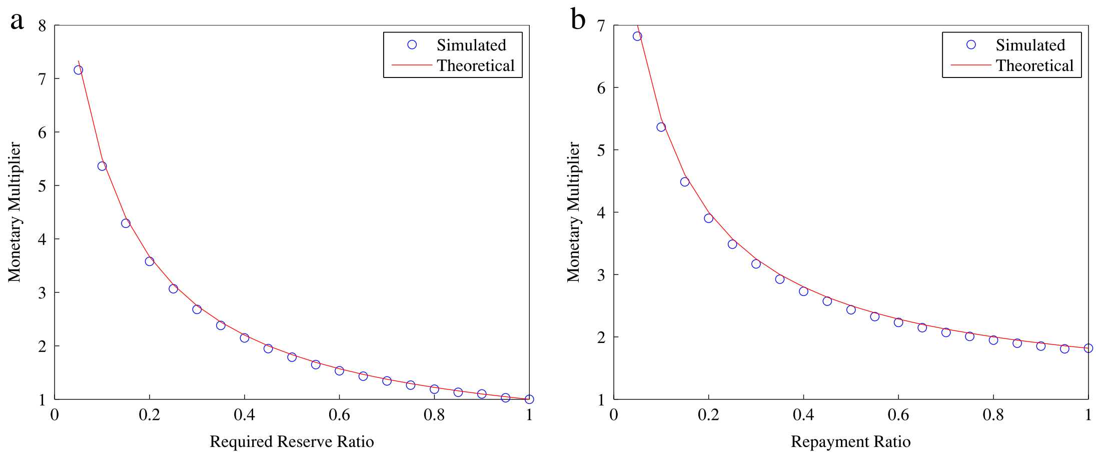

Fig. 7. Money multiplier as a function of (a) γ ($\lambda = 0.1$) and (b) λ ($\gamma = 0.1$).

As for the *Loan Area*, the maximum amount of total expenditure in the system will be the sum of existing money and loans issued (or repayment paid):

$$Y_{\max} = E_{\max} = M + B = M + R. \tag{41}$$

Substituting Eqs. (27) and (35) into (41), we then have

$$Y_{\max} = M + \lambda L = M + \lambda \frac{c_1 - c_1\gamma}{c_1 + c_2\lambda} M. \tag{42}$$

Therefore, the maximum of velocity of money V can be given by

$$V_{\max} = 1 + \lambda \frac{1-\gamma}{1+\lambda}. \tag{43}$$

In the first case, where $\gamma = 0.1$, $\lambda = 1$, $V_{\max}$ is calculated to be 1.45. For the second one, where $\gamma = 0.1$, $\lambda = 0.1$, $V_{\max}$ turns out to be 1.0818. Both estimates are in agreement with the simulation results (1.4520 and 1.0819 in the *Loan Area* of Fig. 8(a) and (b), respectively), which verify the validity of theoretical analysis.

In addition, how the values of γ and λ influence the circulation velocity of money is examined by a computer simulation with $\alpha = 0.8$, $\beta = 0.9$ (a point within the Loan Area), as shown in Fig. 9. We can observe that the velocity goes higher with a smaller reserve ratio γ and a greater repayment ratio λ.

6. Conclusion

In this paper, we investigated the behavioral foundations of the money creation and money circulation processes with an agent-based model. The role of debt is taken into special consideration, and the banking system is characterized as the

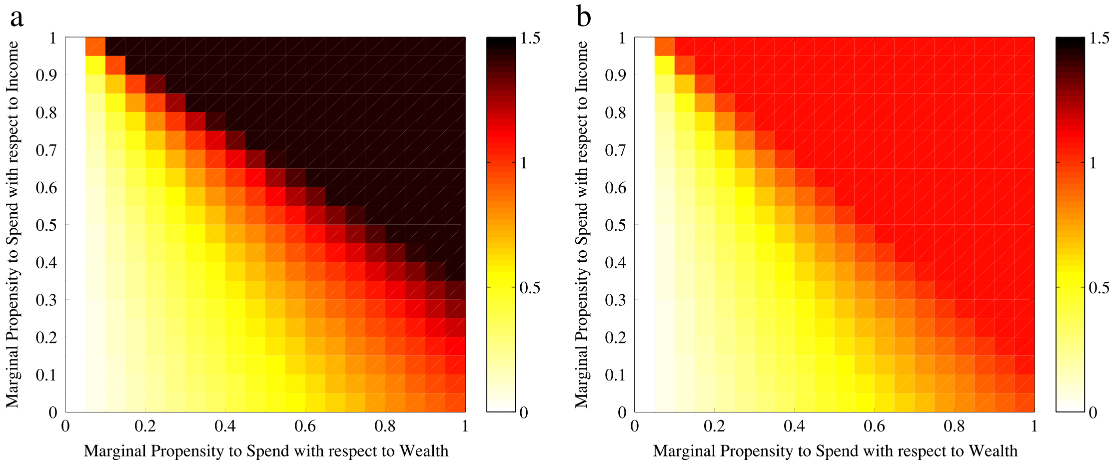

Fig. 8. Velocity of money as a function of α or β with (a) $\gamma = 0.1$, $\lambda = 1$, and (b) $\gamma = \lambda = 0.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

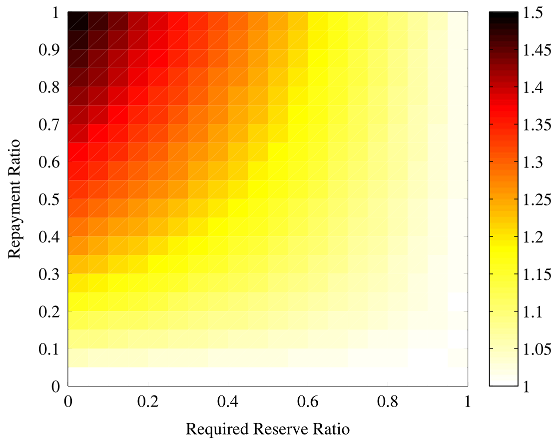

Fig. 9. Velocity of money as a function of γ and λ with $\alpha = 0.8$ or $\beta = 0.9$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

center of money creation and the accelerator of money circulation. With the mean-field approximation method, we obtained the theoretical expressions of the money multiplier and the velocity of money and studied the dependence of these two variables on different parameters through computer simulation. As a result, three major conclusions can be drawn from these analyses. First, we found that the expenditure habits of the public determine the extent to which the economy relies on debt. A clear phase transition between a credit economy and non-credit economy is observed when people vary their propensity to spend. It is also proven that unlike the traditional money creation model, the money multiplier is influenced not only by the borrowing behavior that creates money but also the repayment behavior that destructs money as well. Finally, both money-related and debt-related factors, namely the expenditure habit of agents, the borrowing and repayment behavior of debtors and the monetary policies the banking system is subject to, are found to be the determinants of the velocity of money.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. 2015KJJCA06). We are grateful to Ms. Xuewei Cao for her help in artworks of this paper. We also thank anonymous referees for their pertinent comments and valuable suggestions in revising this paper.

References

[1] B.S. Bernanke, Amer. Econ. Rev. 73 (1983) 257.
[2] B.S. Bernanke, A.S. Blinder, Amer. Econ. Rev. 78 (1988) 435.
[3] B.S. Bernanke, M. Gertler, J. Econ. Perspect. 9 (1995) 27.

[4] J. Moore, N. Kiyotaki, J. Polit. Econ. 105 (1997) 211.
[5] C.E. Borio, P.W. Lowe, BIS Working Paper No. 157, 2014. Available at SSRN: http://ssrn.com/abstract=782324 or http://dx.doi.org/10.2139/ssrn.782324.
[6] S.G. Cecchetti, M.S. Mohanty, F. Zampolli, BIS Working Papers No. 352, 2011.
[7] C.M. Reinhart, K.S. Rogoff, Amer. Econ. Rev. 101 (2011) 1676.
[8] C.M. Reinhart, K.S. Rogoff, A Decade of Debt, Peterson Institute for International Economics, Washington, DC, 2011.
[9] C.M. Reinhart, K. Rogoff, This Time is Different: Eight Centuries of Financial Folly, Princeton University Press, Princeton, 2009.
[10] C.S. Morris, G.H. Sellon, Fed. Reserve Bank Kansas City Econ. Rev. 80 (1995) 59.
[11] A.K. Kashyap, J.C. Stein, D.W. Wilcox, Amer. Econ. Rev. 83 (1993) 78.
[12] M. Schularick, A.M. Taylor, Amer. Econ. Rev. 102 (2012) 1029.
[13] B.M. Friedman, The Changing Roles of Debt and Equity in Financing US Capital Formation, University of Chicago Press, Chicago, 1982.
[14] B.M. Friedman, Macroeconomics, Prices, and Quantities, The Brookings Institution, Washington, D.C, 1983, pp. 161–199.
[15] P. Krugman, New York Times, 2009. http://www.nytimes.com/2009/09/06/magazine/06Economic-t.html (Retrieved 03.03.12).
[16] G.B. Eggertsson, P. Krugman, Quart. J. Econ. 127 (2012) 1469.
[17] W. Godley, M. Lavoie, Monetary Economics, Palgrave Macmillan, Basingstoke, 2007.
[18] S. Keen, Instability in financial markets: Sources and remedies, in: INET Conference, Berlin, Germany, 2012.
[19] S. Keen, Modeling Financial Instability, Working Paper, 2014.
[20] I. Fisher, Econometrica 1 (1933) 337.
[21] H.P. Minsky, Challenge 25 (1982) 5.
[22] K. Brunner, A.H. Meltzer, J. Polit. Econ. 80 (1972) 951.
[23] J.E. Stiglitz, A.S. Blinder, Amer. Econ. Rev. 73 (1983) 297.
[24] T.I. Palley, J. Post Keynesian Econ. 16 (1994) 371.
[25] G. Bernardo, E. Campiglio, Empirica 41 (2013) 381.
[26] D. Braun, Physica A 369 (2006) 714.
[27] R. Fischer, D. Braun, Physica A 321 (2003) 605.
[28] R. Fischer, D. Braun, Physica A 324 (2003) 266.
[29] S. Chen, Y. Wang, K. Li, J. Wu, Physica A 394 (2014) 217.
[30] T.M. Humphrey, FRB Richmond Econ. Q. 79 (1993) 1.
[31] I. Fisher, The Purchasing Power of Money, Macmillan, New York, 1911.
[32] D. Laidler, The Golden Age of the Quantity Theory, Princeton University Press, Princeton, 1991.
[33] P. Bridel, Cambridge Monetary Thought, St. Martin's Press, NewYork, 1987.
[34] M. Friedman (Ed.), Studies in the Quantity Theory of Money, University of Chicago Press, Chicago, 1956.
[35] M. Friedman, A.J. Schwartz, A Monetary History of the United States, 1867-1960, Princeton University Press for NBER, Princeton, 1963.
[36] R.G. Anderson, R.H. Rasche, Federal Reserve Bank of St. Louis Working Paper Series 2001-008, 2001.
[37] M. Gillman, P.L. Siklos, J.L. Silver, Money Velocity with Costly Credit, Department of Economics, University of Melbourne, 1996.
[38] F. Padrini, J. Monetary Econ. 49 (2002) 521.
[39] R.J. Barro, D.B. Gordon, J. Econ. Policy 12 (1983) 101.
[40] W.A. Barnett, H. Xu, Money Velocity with Interest Rate Stochastic Volatility and Exact Aggregation, Department of Economics, Washington University, 1995.
[41] P. Basu, P. Dua, Appl. Econ. Lett. 3 (1996) 581.
[42] Y. Wang, N. Ding, L. Zhang, Physica A 324 (2003) 665.
[43] N. Ding, N. Xi, Y. Wang, Eur. Phys. J. B 36 (2003) 149.
[44] N. Xi, N. Ding, Y. Wang, Physica A 357 (2005) 543.
[45] K. Brunner, Internat. Econom. Rev. 2 (1961) 79.
[46] K. Brunner, A.H. Meltzer, J. Financ. 19 (1964) 240.
[47] J.S. Mill, Principles of Political Economy with some of their Applications to Social Philosophy, George Routledge and Sons, Manchester, 1848.
[48] M.N. Rothbard, Man, Economy, and State, Ludwig Von Mises Institute, Auburn Alabama, 2004.
[49] Y. Wang, H. Qiu, Physica A 353 (2005) 493.
[50] K. Liu, N. Lubbers, W. Klein, J. Tobochnik, B. Boghosian, H. Gould, arXiv:1305.0794 (2013).