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Follow the money: The monetary roots of bubbles and crashes

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Abstract

We propose a reduced form model for the Minskian dynamics of liquidity and of asset prices in terms of the so-called financial accelerator mechanism. In a nutshell, credit creation is driven by the market value of the financial assets employed as collateral in the bank loans. This leads to a self-reinforcing feedback between financial prices and liquidity that we model by coupled non–linear stochastic processes. We show that the resulting dynamics are characterized by a transient super-exponential growth qualifying a bubble regime. Unchecked, this would lead to a finite time singularity (FTS). The underlying singularity expresses the unsustainable dynamics of the corresponding economy and announces a regime change, such as a crash. We propose to describe the dynamics of the crisis by the same coupled non–linear stochastic process with inverted signs, i.e., nonlinear negative feedbacks of value and money on their growth rates. Casting the financial accelerator dynamics into a simple macroeconomic model, we show that the cycle of booms and bursts of financial assets and liquidity determines economic recessions in the form of increasing aggregate default rates and decreasing GDP. Finally, by exploiting the implications of the proposed model on the dynamics of financial asset returns, we introduce a generalized GARCH process, called FTS-GARCH, that can provide an early warning identification of bubbles. Estimating the FTS-GARCH on well-known historical bubble episodes suggest the possibility to diagnose in real-time the presence of bubbles in financial time series.

JEL classification: G01, G17, C53

Keywords: Minskian dynamics, financial bubbles, positive feedback, financial accelerator, generalized FTS-GARCH

1 Introduction

In our current monetary system, liquidity is mainly created by the credit issued by the "banking systems," considered in a broad sense, i.e., including the shadow banking system and other financial institutions behaving de facto like banks. Credit creation is subject to cycles which often coincide with those in economic activity and asset prices. The coincidence of these cycles has already been well documented in the policy-oriented literature (e.g. IMF 2000, BIS 2001). However, only few studies have tried to formally assess the link between liquidity dynamics and the episodes of booms and bursts in the financial markets.

In standard real business cycle models, credit do not have any macroeconomic effects. Moreover, credit aggregates, in particular bank credit that constitutes the closest proxy for money, are usually assumed to be mainly demand-driven (Bernanke and Blinder 1988, Fase 1995, Calza et al. 2001), depending on economic activity and financing costs. However, driven by the implication of asymmetric information, new theoretical approaches have attributed an important role to credit aggregates in the development of the business cycle (e.g. Bernanke et al. 1999 and Kiyotaki and Moore 1997). In particular, Werner (2005) has argued that banks and other institutions acting effectively as banks play a key role in credit creation and that the effect of credit creation depends on how the money injected in the economy is used for. He identifies three types of credit: for production, for consumption and for speculation. Only credit creation for production leads to real GDP growth by increasing the quantity of goods and services. Credit creation for consumption increases GDP but it does so through inflation and not real growth. In this case, more money circulates in the economy but the quantity of goods and services remain the same. Credit creation for financial transactions increases the amount of money in financial markets, leads to asset inflation and is not part of GDP. Excessive credit creation of the last two cases can eventually lead to banking and economic crises.

Because of information asymmetries, agency problems, and incomplete contracts, banks tend to preferably extend loans with real or financial asset as collateral (see Aghion and Bolton (1992), Hart and Moore, (1994, 1998)). Higher collaterals, in fact, reduce the influence of asymmetric information and improve lending conditions. This tendency of the banks for collateralizing loans introduces an important structure in the way money flows into the economy. Hendry (1984) and Muellbauer (1992) showed that availability of bank loans backed with some collateral asset is a major factor in the rise of those asset prices. When increasing new credit is granted for financial investments (so that the collateral of this credit becomes the investments itself) because of the relatively fixed short-run supply of collateral, it will tend to increase the corresponding financial prices. For example, a rise in the amount of issued mortgage will make house prices increase (see e.g. Greiber and Setzer 2008), while easy lending (backed by financial asset) to, for instance, hedge funds, Special Investment Vehicles (SIV), and other financial investors will tend to increase financial asset prices. Because borrowers are constrained in their demand for credit by the values of their collateral assets, an increase in their prices will expand their borrowing capacity augmenting the creation of liquidity which, in turn, increases the collateral prices in a self-reinforcing mechanism. Expansion of asset collateralizing loans push up the price of collateral, allowing bank to expand their collateralized loan books. Similar self-reinforcing interactions have been investigated in the literature which usually referred to them as "financial accelerator" mechanisms (Stein 1998, Bernanke and Gertler 1989, Bernanke, Gertler and Gilchrist 1996, 1999 and Kiyotaki and Moore 1997). The existence of a close link between ample liquidity and bubbles is also reported by practitioners and financial commentators.[1]

In addition, the sustained growth of liquidity and financial prices during the boom phase will create capital gains that will easily repay interest payment and make financial investments appear safe and profitable, thereby reducing the estimated probability of default, the risk premia, and the costs for access to external funds. Because investing in the roaring markets will yield high and consistent capital gains, more and more investments will be re-directed from the real sector to the financial one. During this period, competition among banks for higher profits will induce them to increase their financial leverage. Moreover, firms will also start to invest more on financial instruments and financial acquisitions (e.g. stocks repurchases) and less on physical and productive investments (such as new machineries or R&D). Hence, because of the financial accelerator and investment substitution (from real to financial sector), a growing share of new credit will be channeled to financial investments exerting further pressure on financial prices, which in turn will expand borrowing capacity and hence banks lending to financial investments. As the financialization of the economy increases, the increasing reliance by firms on earnings realized through financial channels progressively marginalize the role of the real economy in the global economic growth and also leads to significant income inequality (Lin and Tomaskovic-Devey, 2011). This has a negative effect on the growth of the real economy, leading eventually to the change of regime from boom to bust.

Here, we propose a reduced form model for this Minskian–type dynamics generated by the financial accelerator. We start by considering credit creation as driven by the market value of the financial asset employed as collateral in bank loans. This leads to a self-reinforcing mechanism between financial prices and liquidity that we model by coupled non–linear stochastic processes. We show that the resulting non–linear dynamics are characterized by super-exponential growth. Unchecked, such super-exponential growth would lead to a finite time singularity. The singularity expresses the unsustainable dynamics of the economy and announces a regime change, such as a crash and/or economic recession. During a financial crisis, a self-reinforcing interaction between liquidity and collateral prices also develops. Being the exact image of the financial accelerator in reverse, we model financial crises with the same coupled non–linear stochastic processes used in the financial accelerator only with the opposite sign. The occurrence of the sign inversion, i.e, the shift of the financial accelerator into a downward spiral, is modeled as a stochastic regime switching process with endogenous transition probabilities.

[1] Mauldin and Tepper (2011), for instance, recently wrote "[...] liquidity has to go somewhere, and emerging markets look like the most likely destination [...] Emerging markets could easily be the next bubble.".

Casting the financial accelerator dynamics into a simple macroeconomic model, we show that the relations between liquidity and asset prices have important consequences on general macroeconomic variables. Fluctuations in asset prices and liquidity heavily impact on borrowing capacity, cost of external funds, and probability of default, hence, strongly affecting aggregate demand and real GDP.

Finally, exploiting the implication of the super-exponential dynamics of prices on log returns, we propose a simple modification of the standard GARCH model, that we coin FTS-GARCH, with which it becomes possible to investigate the presence of finite time singularity behaviors, and hence bubbles, in financial series. We apply this procedure to various financial time series during a bubble period and find evidence for super-exponential behaviors.

Our analysis is related to the literature on the credit channel view of monetary policy transmission and on that of non–linear feedback mechanisms leading to financial bubbles and bursts. Some recent papers addressing these topics (mainly from an empirical point of view) include Adrian and Shin (2008) on the procyclical nature of leverage, Benmelech and Bergman (2009) on credit traps, and Sornette and Woodard (2009), Johansen and Sornette (2010), and Lin Ren and Sornette (2009) on the diagnostic and modeling of financial bubbles with super-exponential price dynamics.

The paper is organized as follows. Section 2 describes the coupled stochastic processes induced by the financial accelerator mechanism for the dynamics of financial prices and financial investments. Section 3 describes the real sector part of the macroeconomic model. Section 4 illustrates the dynamic behavior of the model by means of Monte Carlo simulations. Section 5 proposes the modified FTS-GARCH model to identify the presence of bubble and applies it on empirical data. Section 6 summarizes the implications of the model for economic stability and policy decisions. The Appendix presents the exact solution of the finite-time singular behavior of a deterministic toy version of the model.

2 Liquidity and asset prices: A reduced form model for the financial accelerator

2.1 Reduced form model of money channeled to the financial market and financial prices

Let us denote M^F, the amount of money channeled to the financial market, and M^R, the amount absorbed by the real sector. We model the self-reinforcing interaction between money used for financial investments M^F and financial prices P with the following coupled non-linear stochastic system:

$$dM_t^F = \alpha P_t M_t^F dt + \sigma_{M^F} M_t^F dW_t^{M^F}. \quad (1)$$

$$dP_t = \beta M_t^F P_t dt + \sigma_{t,P} P_t dW_t^P \quad (2)$$

where dW^{M^F} and dW^P are two independent Brownian motions, σ_{M^F} and $\sigma_{t,P}$ are the volatilities of M^F and P respectively, β and α are scalar parameters. To take into account the well known presence of heteroskedasticity in the volatility of financial assets, we assume $\sigma_{t,P}$ to follow a stochastic process (i.e. P_t is a stochastic volatility process).

The cross product $\alpha P_t M_t^F$ in the equation for dM_t^F captures the following channel of growth: increases in collateral values P_t lead to greater lending due to the reduction in financial constraints, hence increasing the amount of money channeled to the financial sector. The cross product $\beta P_t M_t^F$ in the equation for dP_t embodies that a greater volume of lending, in turn, increases liquidity in the corporate and household sector, which can then bid more aggressively when acquiring collateral assets (Shleifer and Vishny, 1992), thus pushing prices up.

2.2 Approximated solution and the concept of "finite-time singularity"

The feedback between financial prices P and money channeled to financial market M^F lead to a non–linear dynamics with super–exponential behavior, as will be shown here and the following sections.

Appendix A gives the exact solution of equations (1) and (2) for the special case $\sigma_F = \sigma_P = 0$, i.e. for the purely deterministic case. In the presence of positive feedbacks of financial price on money ($\alpha > 0$) and of money on price ($\beta > 0$), the solution exhibit a finite-time singularity (FTS), i.e., the standard exponential growth obtained in the standard case where the two equations (1) and (2) are decoupled (i.e., P_t is fixed to a constant P in (1) to obtain $M_t^F = M_0^F \cdot \exp[\alpha P \cdot t]$ or M_t is held equal to a constant M in (2) to obtain $P_t = P_0 \cdot \exp[\beta M \cdot t]$) is replaced by a super-exponential growth which is unsustainable: divergences appear in finite time, signaling a change of regime.

Such finite-time singularities are not monsters that exist only in the imagination of mathematicians. They reflect the deep fact that positive feedback may often lead to solutions that exit only over a finite duration, hence forcing to take into account new mechanisms and processes that have to come into play as the singularity is approached. The mathematical singularity is the signature that positive feedbacks can only dominate the dynamics over a finite time interval, and the finite-time singularity just announces the coming change of regime to a new behavior dominated by different controlling mechanisms. The insistence by economists to work only with models for which existence of solutions for *all times* can be proved has prevented the consideration of models with FTS, since the solution stops to exist beyond the critical time. We believe that this insistence on existence of solutions for *all times* is misplaced because it removes by construction a rich class of transient solutions that exemplify the dynamics of instabilities that may be a non-negligible component of the behavior of financial markets.

In endogenous growth economics, the positive feedback of human population on R&D and productivity leading to increased carrying capacity has been noticed by Kremer (1993), reduced in its essence to the simple equation

$$\frac{dp}{dt} = R \cdot p(t)^{1+\delta} \ ,. \tag{3}$$

For $\delta = 0$, there is no positive feedbacks and the solution is the standard Malthusian exponential growth. For $\delta > 0$, the solution of equation (3) reads

$$p(t) = \frac{p_0}{(1 - t/t_c)^{1/\delta}} \qquad \text{iff } \delta > 0 \ , \tag{4}$$

where

$$t_c = \frac{1}{\delta R} \, p_0^{-1/\delta} \ , \tag{5}$$

with the initial condition $p(t = 0) = p_0$. Here, the critical time t_c at which the solution diverges is determined from the parameters R and δ of equation (3) and the initial population p_0. As Appendix A shows, the positive feedbacks of price on money and money on price leads exactly to this reduced form $dM/dt \simeq M^{1+\delta}$, with $\delta = 1$. Note also that solution (4) with (5) recovers the standard exponential solution $p(t) = p_0 e^{Rt}$ in the limit $\delta \to 0^+$ as it should. This can be seen by using the expansion $(1 - t/t_c)^{-1/\delta} = \exp[-(1/\delta) \ln(1 - t/t_c)] \simeq \exp[(1/\delta)(t/t_c)] = e^{Rt}$ for $t \ll t_c \to +\infty$ for $\delta \to 0^+$.

The singular solution (4) was first discussed by von Foerster et al. (1960) (see Umpleby (1990) for assessments of the relative merits of the "natural science" versus the "demographic" approach, Kremer (1993) for an economic underpinning, and Johansen and Sornette (2001); Korotayev et al. (2006) for extensive generalizations). In ecology, the positive correlation between population density and the per capita population growth rate at the origin of the FTS behavior (4) is known as the Allee effect, see for instance Stephens et al. (1999). More generally, Allee discovered the existence of an often present positive relationship between some components of individual fitness and either numbers or density of conspecifics. The Allee effect is usually used to refer to the self-reinforcing feedbacks that promote accelerate extinction of species, that can be modeled by finite-time crossing of zero, see Yukalov et al. (2009). Goriely (2000) provides a rigorous mathematical framework with a generalized version of equation (3), where the right hand side is replaced by an arbitrary polynomial of $p(t)$. The use of the mathematics of FTS to describe and diagnose changes of regime is not new. For instance, we refer to Johansen and Sornette (2001); Sornette (2003) for population dynamics and financial markets, Sammis and Sornette (2002) for applications to engineering failures and earthquakes, Sornette (2002, 2006) for a large variety of systems, Dakos et al. (2008) for climate systems, and Scheffer et al. (2009); Biggs et al. (2009); Drake and Griffen (2010) for environmental systems. These authors applied the concept of dynamical phase transitions and FTS to different systems exhibiting a bifurcation, crisis, catastrophe or tipping point, by showing how specific signatures can be used for advanced warnings.

2.3 The model for financial crises

During a financial crisis, the interaction between liquidity and collateral prices also takes place according to a positive feedback loop as follows. A significant reduction in the value of the assets backing high leveraged positions leads to margin calls that force the borrowers to fire sale the asset, which in turn tends to push its value down. This fall in the asset prices reduces the value of the collateral backing the previous leveraged credit boom. A reduction in collateral prices will then contract borrowing capacity, reducing lending and liquidity, which in turn decreases the collateral prices further and so on (this process is well described, among others, in Adrian and Shin 2008, Benmelech and Bergman 2009, and Acharya and Richardson 2009).

Hence, the very same mechanism that gave rise to the financial accelerator during the boom phase is at work during the crash phase, with just a different sign in its action. Being basically the same process as the financial accelerator with only a reversed direction, we model financial crises with the same dynamical equations (1) and (2) with a change of signs of the parameters α and β. For $\alpha, \beta > 0$, positive feedbacks lead to reinforcing accelerated growth of money supply and prices. For $\alpha, \beta < 0$, negative feedbacks lead to a correlated decrease of the money supply and prices. The burst of a financial bubble occurs in our model when the sign of the parameters α and β of the coupled equation system (1) and (2) shift from positive to negative.

This change of regimes can be effectively modeled with a regime switching model with endogenous transition probabilities. We assume that both parameters α and β are controlled by an unobserved aggregate state parameter that can live in two possible states $s_t = \{H, L\}$ (boom and recession). Each parameter can assume two values: $\alpha(s_t = H) \equiv \alpha_H > 0$, $\alpha(s_t = L) \equiv \alpha_L < 0$ and $\beta(s_t = H) \equiv \beta_H > 0$, $\beta(s_t = L) \equiv \beta_L < 0$. In order to capture the observed asymmetry in the boom–burst cycle (i.e. the downhill being typically steeper than the uphill), we take $|\alpha_H| < |\alpha_L|$ and $|\beta_H| < |\beta_L|$. Alternately, we could generate such asymmetry by introducing the negative effect of aggregate corporate defaults (whose dynamics will be described later) in the equation of M^F.

For the dynamics of the state variable, we follow the description of regime shifts based on a non–linear dynamical process (Yukalov et al., 2009), which can provide an endogenous underpinning of Markov Chain switching models (Hamilton, 1989; Hamilton and Raj, 2002). We then propose s_t to be given by the sign, positive for H and negative for L, of the variable ξ_t that follows the non–linear stochastic process:

$$d\xi_t = (a\xi_t - bx_t\xi_t^2 - c\xi_t^3)dt + \sigma_\xi dW_t^\xi \quad \text{and} \quad \begin{cases} s_t = H & \text{if} \quad \xi_t \geq 0 \\ s_t = L & \text{if} \quad \xi_t < 0 \end{cases} \tag{6}$$

Hence, ξ_t is an asymmetric bistable process with a dynamically changing asymmetry given by the triggering variable x_t. The triggering variable x_t can be function of different quantities such as (i) the degree of mispricing ($x_t = f(P_t/F_t)$ with F representing the fundamental value of the asset), (ii) the degree of misallocation of loans ($x_t = f(M_t^F/M_t^R)$), or (iii) the degree of financialization of the economy ($x_t = f(P_t/Y_t)$ with Y_t the real GDP).

The dynamics of ξ_t has, therefore, two attractors (one positive and one negative) so that the stochastic trajectory can remain for quite a long time inside one of the corresponding trapping regions and then jump stochastically to the other one. Both the mean time spent in each basin of attraction and the probability of switching from one basin to the other depends on the asymmetric coefficient x_t which represents the degree of mispricing or financialization of the economy.

For instance, in the case of $x_t = \log(P_t/F_t)$, when the economy is in the boom phase (with $\xi_t > 0$ and hence $s_t = H$), the further the market price P_t drifts away from the fundamental price F_t, the more asymmetric the distribution of the process will be and the higher the probability of switching to the negative attractor (with $\xi_t < 0$ and $s_t = L$). Once in the new regimes, a strong asymmetry in the opposite direction will be needed for the system to switch again to the other attractor. In this way, we are able to also model the typical overshooting of financial prices below the fundamental values during a recession.

In this case, it seems also sensible to assume that the Brownian innovation dW_t^ξ is highly correlated with (if not exactly the same as) the innovation dW_t^P in (2) so that a stochastic switching from the positive state H to the negative L is triggered by a negative shock in P_t and vice versa for the switching in the opposite direction.

3 The stylized macroeconomic model

In this section, we briefly describe the equations of motion of the other most relevant macroeconomic variables of our economy: the aggregate corporate default rate and the other variables of the real sector, which are the amount of money directly channeled to GDP related investments and the real GDP.

3.1 The dynamics of aggregate corporate default rate

Let us define with B_t the total amount in value of the firm's bankruptcies (loss given default) occurring during the measurement period (e.g., one quarter or one year). Denoting $M_t = M_t^F + M_t^R$ the total money supply, sum of the money flowing both to the financial and to the real economy sectors, the rate of growth m_t of the total money supply M_t is simply

$$m_t = \frac{1}{M_t}\frac{dM_t}{dt} \,. \tag{7}$$

Denoting R_t the average nominal interest rate, the dynamics of the aggregate firm's bankruptcy or default can be modeled, in reduced form, as depending on the differential between R_t and m_t since the first variable determines the weight of the debt service while the second indicates the dynamics of the overall liquidity circulating in the system. We thus write

$$dB_t = (R_t - m_t)B_t dt + \sigma_B B_t dW_t^B \ , \tag{8}$$

where σ_B is the volatility of the bankruptcy process. Equation (8) simply says that, if the interest rate is larger than the growth rate of money, the aggregate rate of bankruptcy tends to increase because there will not be enough money to repay all the existing debts (principal plus interests). In contrast, when the interest rate falls below the growth rate of money, the money circulating will be larger than the one needed to repay the debts and hence the aggregate default rate will tend to decline. This relationship between interest rate and default risk is supported by a study of firm level corporate default data in the United States between 1982 and 2008 (Kapling et al., 2009) that finds significant negative contemporaneous correlations between the changes in short interest rates and aggregate default rates, with a particularly strong relationship around financial crises.

Equation (8) represents an additional mechanism (and arguably a dominant one during some periods, such as during recessions) that operates as a transmission process for monetary policies: ceteris paribus, when interest rates are reduced, bankruptcies in the economy decline while when interest rates are raised the number of bankruptcies significantly increase slowing down the economic growth. We can call this transmission mechanism of monetary policy *the default transmission channel*.

3.2 The real sector

Unlike the dynamics of the amount of money channeled to the financial market M^F, the dynamics of the amount of money absorbed by the real sector M_t^R has a growth rate negatively related to excessively high financial prices P, because of the effect of investment substitution (from real to financial sector) that is present when financial markets are soaring. This is captured by the following dynamics for M_t^R:

$$dM_t^R = -\theta \frac{P_t}{F_t} M_t^R dt + \sigma_{M^R} {M^R}_t dW_t^{M^R} , \tag{9}$$

where θ is a positive parameter and F_t is the fundamental or equilibrium price of P_t. During the boom phase, the amount of money absorbed by the financial market will grow at a much faster rate than the amount of money directed to the real sector. Financial prices will also rise faster than other prices.

In order to complete the modeling of our stylized economy, we also specify the dynamics of the real GDP which is assumed to follow:

$$dY_t = g M_t^R Y_t dt - b \ dB_t + \sigma_Y Y_t dW_t^Y . \tag{10}$$

where g and b are positive coefficients. Equation (10) simply says that GDP tends to increase with the amount of money channeled to the real sector M_t^R, while it tends to decrease with rises in bankruptcies (which tends to destroy or at least under-utilize technological know-how and human capital).

3.3 The full model

Summarizing, the full model is then composed by the following system of dynamic coupled equations:

$$\begin{aligned}
\text{money to financial sector:} \quad dM_t^F &= \alpha(s_t)\, P_t M_t^F dt + \sigma_{M^F} M_t^F dW_t^{M^F} && (11) \\
\text{financial prices:} \quad dP_t &= \beta(s_t)\, M_t^F P_t dt + \sigma_{t,P} P_t dW_t^P && (12) \\
\text{money to real sector:} \quad dM_t^R &= -\theta \frac{P_t}{F_t} M_t^R dt + \sigma_{M^R} {M^R}_t dW_t^{M^R} && (13) \\
\text{aggregate bankruptcies:} \quad dB_t &= (R_t - m_t) B_t dt + \sigma_B B_t dW_t^B && (14) \\
\text{real GDP:} \quad dY_t &= g M^R Y_t dt - b\; dB_t + \sigma_Y Y_t dW_t^Y && (15)
\end{aligned}$$

with F_t the fundamental value, $m_t = (1/M_t) dM_t/dt$, $M_t = M_t^F + M_t^R$ and $\alpha(s_t)$ and $\beta(s_t)$ taking two values $\alpha(s_t = H) \equiv \alpha_H > 0$, $\alpha(s_t = L) \equiv \alpha_L < 0$ and $\beta(s_t = H) \equiv \beta_H > 0$, $\beta(s_t = L) \equiv \beta_L < 0$, depending on the sign of variable ξ_t following the non–linear stochastic process (6).

4 Model dynamics

To gain some intuition on the dynamical properties of the non–linear stochastic processes (11-15), we illustrate the dynamic behavior of the financial accelerator and of the stylized macroeconomic model by means of Monte Carlo simulations. Note that M_t^F and P_t are just functions of each other, so that the Monte Carlo simulations can be decomposed into two steps: (i) generation of the trajectories for M_t^F and P_t with (11) and (12), which then (ii) impact that of M_t^R (13), which, together with M_t^F, controls the dynamics of both B_t and Y_t, via definition (14) and (15).

Insert Figure (1) about here

Figure 1 shows the logarithm of money M^F and of the price P scaled by GDP as a function of time represented in linear unit. A straight line in such semi–log scale qualifies a standard exponential growth with constant growth rate. The upward curvature exhibited by the M^F and P dynamics clearly indicates the existence of a super-exponential growth culminating into a finite-time singularity. The behavior of these simulated trajectories are qualitatively similar to those empirically observed in many countries during the periods preceding the burst of a financial bubble. As a visual comparison, Figure 2 shows the evolution of various stock market indices during periods commonly identified as bubbles.

Insert Figure (2) about here

In Figures 3 and 4, we show the dynamical properties of the full model by simulating the whole system (11-15) of dynamic equations. Again, the behavior of these simulated trajectories are qualitatively similar to those observed empirically in many countries during the periods before and after the burst of a financial bubble.

Insert Figures 3 and 4 about here

Typical of Minskian dynamics, during the growing phase of the bubble, financial markets are soaring, there is easy and cheap access to credit, firms are making profit (though often a large part of it is through financial activities), bankruptcy rates are low and the whole economy seems safely growing. An ideal financial and economic new world seems to be born in which the old bounds and constraints seem not to hold anymore.

The accelerating growth of the financial prices and financial investments makes the financial sector grows explosively. This is not sustainable since the increasing burden of debt service subtracts resources to the aggregate demand and tends to concentrate wealth which also reduces the aggregate demand. In Minskian terms, the safety margins of the indebted units are reduced and the overall fragility of the financial system increases; growth and fragility then progress together. Because of a weakening aggregate demand and increasing leverage, the current cash flows of many agents reach the point where they are insufficient for servicing the debt, leading to a strong increase of the probability of bankruptcies that can reach epidemic proportions.

When the financial prices stop growing and start declining, the very same self-reinforcing interaction between money and collateral prices that gave rise to the financial accelerator will continue to operate but now with the opposite sign. A reduction in collateral prices causes margin calls, fire sales and a contraction of contract borrowing capacity that reduces the creation of money which, in turn, depresses the collateral prices and so on. Moreover, another effect becomes dominant in the economy, namely the sharp increase in the default rate induced by the contraction of money supply, as modeled by equation (1). In fact, the key differential $R_t - m_t$ in equation (8) becomes positive making the bankruptcy rate increases as a result of the liquidity evaporation and unbearable interest costs.

Summarizing the full boom-burst cycle, during the boom phase, financial prices and liquidity both rise at an accelerating pace as described by equations (1, 2). The losses due to defaults shrink thanks to the abundant liquidity pumped into the system, as modeled by equation (8). The GDP tends to follow the general euphoric climate by trending upward. Eventually, the real economy starts to suffer from the competition with the highly profitable and seemingly safe investment financial products. Indeed, the level of investment in the real economy starts to decline once the speed of the financial price growth accelerate, as given by equation (9). When the fragility of this tumultuous growth become apparent, financial prices stop to rise and start to invert their trends, triggering the vicious cycle of the crash, whose trigger is controlled by the dynamics of a state variable given by equation (6). Due to the drying up of liquidity in the system, many corporations start to suffer from funding difficulties and the bankruptcy rate rapidly increases according to the dynamics expressed in equation (8). This, in turn, deeply depresses aggregate demand and GDP, transmitting the crises to the real sector according to equation (10).

To conclude, the proposed model provides a first quantitative framework to theoretically describe the empirically found strong relation between credit and economic activity cycles. For instance, Alessi and Detken (2009) compare a total of 89 early warning indicators for high-cost asset price boom/bust, finding as the best indicators the global M1 gap and the global private credit gap (defined as the weighted average across countries of detrended private credit to GDP ratios).[2]

5 Empirical analysis: Bubble detection

In this section, we exploit the implications of the proposed model on the dynamics of financial asset returns to suggest a procedure for identifying the presence of bubbles (defined as period of faster than exponential growth) in both a static (ex-post identification) and dynamic (real-time detection) framework.

5.1 The FTS-GARCH model

As Appendix A shows, the positive feedbacks of price on money and money on price leads to a finite time singular (FTS) dynamics in which both M and P follow a self–reinforcing dynamics that can be captured by the simple equation of the type $dX/dt \simeq X^{1+\delta}$, with $\delta > 1$. This means that during a bubble and a crash, price growth and decline depend on price levels. Defining $r_t \equiv \ln(P_t) - \ln(P_{t-1})$, a FTS dynamics then implies that the conditional mean of r_t is a function of the price level P_t. Since the GARCH process provides a reasonable description of the time changing volatility that characterizes financial asset returns, we propose to model r_t as a GARCH model in which a FTS dynamics in the conditional mean return is added. The resulting FTS-GARCH model capturing these two dynamical features of financial prices then reads:

$$r_t = \mu + \gamma(s_t)P_{t-1} + \epsilon_t \quad (16)$$

$$\sigma^2_{t,P} = \omega + \alpha_P \epsilon^2_{t-1} + \beta_P \sigma^2_{t-1,P} \quad (17)$$

[2]This interesting result has also been cited by the President of the European Central Bank, Jean-Claude Trichet, during his speech on the occasion of the Susan Bies Lecture, held at Evanston, Illinois on 27 April 2010: "Research conducted at the ECB and elsewhere points to a link between boom-bust financial cycles and the evolution of broad measures of money and credit. For example, the ratio of global credit to global GDP offers an indication of nascent financial stress. The departure of this ratio from its historical trend offers a signal of emerging financial imbalances and asset price misalignment.".

where $\epsilon_t = \sigma_{t,P} z_t$, $\gamma(s_t)$ is a state dependent parameter taking positive (during bubbles) or negative (during crashes) values depending on the state of the economy s_t, and ω, α_P, β_P are positive parameters.

We test $\gamma(s_t) = 0$ under the null hypothesis that the price process P_t is a random walk with drift or a trend stationary process, cases in which the usual t-statistics have standard asymptotic distribution (Hamilton 1994).[3] We then interpret the rejection of the null hypothesis of $\gamma(s_t) = 0$ as an evidence of the presence of a FTS dynamics and thus of a bubble.[4]

[Insert Table 1 about here]

We then investigate the existence of a FTS dynamics (identifying the presence of a bubble) in various assets during periods related to well-known bubble growths (when $\gamma(s_t)$ is expected to be positive): the S&P 500 index over two different periods 1968-2000 and 1977-1987, the NASDAQ Composite index from 1988 to 2000, the German DAX index and the STOXX 50 index from 1994 to 1998, and the NIKKEI 225 index from 1973 to 1990.

Table 1 reports the results of the estimation of the FTS-GARCH model defined by Eq. (16), (17). The t-statistics for the FTS parameter γ are all significant at the standard 95% confidence level. The rejection of the null hypothesis $\gamma = 0$ suggests the presence of a FTS dynamics in these series during the considered time spans. These indices were indeed characterized by well-known strong bubbles during these periods (as for instance, the bubble of 1980-1987 ending in the Oct. 1987 crash for the S&P 500 index and the ICT bubble from 1995 to 2000 for the NASDAQ index ending in the April 2000 crash). Hence, the identification of a significant FTS γ parameter suggests the possibility to statistically diagnose the presence of bubbles in financial time series.

The ability of the FTS-GARCH model to capture the main dynamic feature of a self-reinforcing process makes it also useful for the estimation of our reduced form model for the financial accelerator. Indeed, one could estimate the parameters in equations (1) and (2) with an indirect inference approach with the FTS-GARCH as an auxiliary model. Hence, by estimating a process such as the FTS-GARCH model on both the empirical and simulated data and minimizing the discrepancy between the two sets of parameters of the auxiliary model, one can get estimates for the parameters in equations (1) and (2). The same indirect inference approach could be analogously used to estimate the parameters of the other equations in the system by using their discrete time counterparts as auxiliary models. Results obtained by this estimation approach will be reported elsewhere.

[3] Standard estimation methods for the γ parameter remain valid in presence of general non-stationary regressors with stationary error terms: the estimator becomes super-consistent, i.e. the speed of convergence to the asymptotic distribution becomes faster than the standard $\sqrt{n}$.

[4] To our knowledge, a formal test which directly test the null hypothesis of FTS dynamics in this setting is not available in the literature. The development of such a test is beyond the scope of the present paper and will be the subject of future studies. In a different direction, Lin et al. (2009) and Lin and Sornette (2011) have proposed non-linear transformations to map the FTS dynamics into a stationary ones on which more standard tests can be applied.

5.2 Real-time bubble detection

In order to construct an indicator for the existence of bubbles working in real time, we propose to investigate the dynamics of the significance of the FTS γ parameter during the build up of a bubble. The dynamics of the t-statistics is obtained by estimating the FTS-GARCH model over a daily moving window. Using windows with longer time span would permit to identify bubbles with long build-up periods while using windows with shorter time spans tend to pick up bubbles that develop over shorter periods. Given that the build-up period for any given bubble is specific to it and varies from bubble to bubble, we select the length of the moving window for our analysis to be approximately close to the time spanned by bubble.

Figures 5-9 show the evolution of the t-statistics for the FTS parameter γ for the various series together with their (rescaled) asset prices (also magnifying their evolutions around the peak of the bubbles). Comparing the dynamics of the t-statistics with that of the corresponding price, we notice that, as the bubble develops, the t-statistics tends to steadily increases becoming more and more significant towards the peak of the bubble, where it reaches its maximum value (often in advance of the maximum of the prices). These results, then suggest that, by looking at the evolution of the significance of the FTS γ parameter, it could be possible to identify the build up of a bubble in real-time.

6 Conclusions

We have develop a reduced form model embodying the Minskian mechanism of financial bubbles, based on feedback loops between collateral values, lending, and liquidity, leading to what is often referred to as the financial accelerator. In a nutshell, increases in collateral values permit an increase in lending due to the relaxation of borrower financial constraints. Greater lending, in turn, increases liquidity in the financial sector which tends, through more aggressive bids, to further increase the values of the collateral assets.

The financial bubble fueled by the growing credit expansion through the financial accelerator leads to super-exponential dynamics of the financial variables that would end, if unchecked, into finite-time singularities, i.e. explosive behavior in finite time. At the same time, investments in the real sector do not only grow slowly but progressively become vampirised (through the substitution effect) by the financial sector. The different growth rate between real and financial sectors make the two corresponding dynamics diverge over time. At some point (the Minsky moment), the shrinking basis of the real sector will cease to sustain the growing pyramid of the financial sector and the system will suddenly collapse. The bigger the bubble, the higher is the potential fall of the global economy.

Financial bubbles and crashes are here viewed as a systematic byproduct of the way money is issued and channeled in the economy. The accelerating behavior of the money dynamics imposes an ever growing increase of the amount of money in the system. Moreover, instead of openly manifest itself through inflation in goods and services prices, the excess of money creation disguises itself in the prices of financial assets which instead covertly accelerate (through the financial accelerator mechanism) the growth of money until the complete collapse of the system occurs.

Moreover, by exploiting the implications of the model on the financial asset time series, we showed that the proposed model can help identifying, not only ex-post but also in real-time, the presence of a bubble in a financial time series, representing a possible highly valuable tool for regulators and supervising authorities.

Appendix

A Finite-time singular behavior of a deterministic toy version of the model

We consider the system of equations:

$$\frac{dM}{dt} = \alpha MP , \tag{18}$$

$$\frac{dP}{dt} = \beta MP . \tag{19}$$

derived from (1) and (2) in the deterministic case $\sigma_F = \sigma_P = 0$.

Equations (18) and (19) imply

$$\frac{1}{\alpha}\frac{dM}{dt} = \frac{1}{\beta}\frac{dP}{dt} = MP , \tag{20}$$

and therefore

$$P(t) = \frac{\beta}{\alpha}(M(t) - c) , \tag{21}$$

where the constant c is determined from the values P_0 and M_0 of $P(t)$ and $M(t)$ at $t = 0$:

$$c = M_0 - \frac{\alpha}{\beta}P_0 . \tag{22}$$

Putting expression (21) in (18) yields

$$\frac{dM}{dt} = \beta M(M - c) . \tag{23}$$

This equation (23) can be integrated exactly and its solution is

$$M(t) = \frac{c}{1 - Ae^{\beta ct}} , \tag{24}$$

where A is a constant of integration such that $c/(1 - A) = M_0$. Note that $\beta c = \beta M_0 - \alpha P_0$, which can be positive or negative.

Consider $\alpha > 0$ and $\beta > 0$, corresponding to local positive growth in equations (18) and (19). Three types of finite-time singularity (FTS) exist.

1. **Finite-time singularity (FTS) of type 1**: $c > 0$, that is, $\beta M_0 > \alpha P_0$. Then, $0 < A < 1$ and $M(t)$ exhibits a FTS, i.e., it explodes to infinity in finite time. Indeed, there is a finite time $t_c = -(\beta c)^{-1} \ln A$, which is strictly positive, such that the denominator $1 - Ae^{\beta ct_c}$ vanishes. Close to this time, we expand the exponential to obtain

$$1 - Ae^{\beta ct} = 1 - Ae^{\beta c(t_c+t-t_c)} \approx 1 - Ae^{\beta ct_c}\left[1 + \beta c(t - t_c)\right] = \kappa(t_c - t) , \quad \text{with } \kappa \equiv \beta cAe^{\beta ct_c} . \tag{25}$$

   This implies that, close to the critical point t_c, its behavior is $M(t)$ is given to leading order by the hyperbolic divergence

$$M(t) \simeq \frac{c}{\kappa} \cdot \frac{1}{t_c - t} . \tag{26}$$

   This retrieves the solution $\sim 1/(t_c - t)$ directly obtained from $dM/dt = \beta M^2$, which is derived from (23) by neglecting the c term for large M in the rhs $\beta M(M - c)$.

2. **Finite-time singularity of type 2**: $c < 0$, that is, $\beta M_0 < \alpha P_0$. Then, $A > 1$ and $M(t)$ also exhibits a finite-time singularity as it is of the form

$$M(t) = \frac{|c|}{Ae^{-\beta|c|t} - 1} . \tag{27}$$

   The singularity occurs when the denominator crosses 0 and develops with the same hyperbolic shape as (26).

3. **Finite-time singularity of type 3**: $c = 0$, that is, $\beta M_0 = \alpha P_0$. Then, we need to come back to expression (23) rather than using the generation solution form (24), which reads

$$dM/dt = \beta M^2 \ . \tag{28}$$

Its solution is exactly

$$M(t) = \frac{1}{\beta} \cdot \frac{1}{t_c - t} \ , \tag{29}$$

also a FTS at the critical time $t_c = 1/\beta M_0$ determined by the initial condition $M(t = 0) = M_0$.

For $\beta < 0$ and $c < 0, \beta c > 0$, the solution has no FTS and the solution is

$$M(t) = \frac{|c|}{(1 + (|c|/M_0))e^{\beta ct} - 1} \ , \tag{30}$$

which decays exponentially to 0 at large times.

Note that $\beta < 0$ and $c > 0$ is possible only if $\alpha < 0$ and P_0 sufficiently large. In this case, $M(t)$ decreases from $M_0 > c$ to c at large times, according to

$$M(t) = \frac{c}{1 - Ae^{-|\beta|ct}} \ , \tag{31}$$

where $0 < A < 1$ is such that $c/(1 - A) = M_0$. The case $M_0 < c$ can be treated similarly.

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$$r_t = \mu + \gamma(s_t) P_{t-1} + \sigma_{t,P}\epsilon_t$$

$$\sigma^2_{t,P} = \omega + \alpha_P r^2_{t-1} + \beta_P \sigma^2_{t-1,P}$$

	mean eq.		variance eq.		
	μ	γ	ω	α_P	β_P
S&P 1968-2000	$2.50 \cdot 10^{-2}$	$6.30 \cdot 10^{-5}$	$9.15 \cdot 10^{-3}$	0.067	0.924
	(1.98)	(2.01)	(7.96)	(46.5)	(332)
S&P 1977-1987	$-4.91 \cdot 10^{-2}$	$5.87 \cdot 10^{-4}$	$8.58 \cdot 10^{-3}$	0.038	0.950
	(-1.13)	(2.11)	(3.34)	(7.51)	(135)
NASDAQ 1988-2000	0.029	$6.18 \cdot 10^{-5}$	$1.90 \cdot 10^{-2}$	0.108	0.877
	(1.19)	(3.16)	(7.92)	(14.29)	(103.69)
DAX 1994-1998	-0.142	$8.62 \cdot 10^{-5}$	0.032	0.081	0.891
	(-1.52)	(2.67)	(3.75)	(5.00)	(43.24)
STOXX 1994-1998	-0.127	$1.20 \cdot 10^{-4}$	$1.26 \cdot 10^{-2}$	0.064	0.918
	(-1.68)	(2.89)	(3.16)	(5.74)	(65.52)
NIKKEI 1973-1990	0.028	$3.79 \cdot 10^{-6}$	$3.21 \cdot 10^{-2}$	0.226	0.748
	(1.80)	(3.48)	(10.57)	(43.66)	(90.74)

Table 1: Estimation of the FTS-GARCH model defined in Eq. (16), (17) for the S&P 500 index from December 1968 to February 2000 (7871 daily observations) and from January 1977 to August 1987 (2691 daily observations), the NASDAQ Composite index from March 1988 to March 2000 (3036 daily observations), the DAX index from January 1994 to June 1998 (1111 daily observations), the STOXX 50 index from from January 1994 to June 1998 (1178 daily observations), and the NIKKEI 225 index from January 1973 to January 1990 (4203 daily observations). t-statistics are in parenthesis.

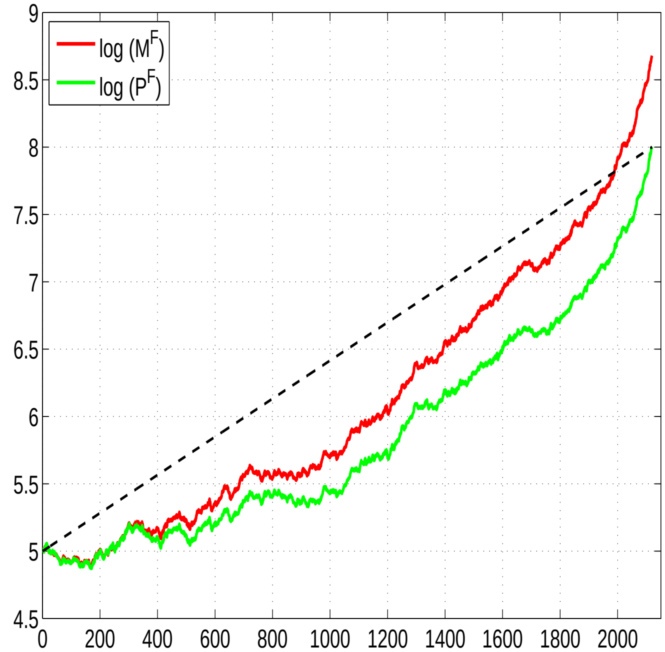


Figure 1: Simulated trajectories for the logarithms of the money channeled to the financial sector M^F (dark line) and of the financial price P (light line). In such a representation, a constant growth rate corresponding to a standard exponential growth is qualified by a straight line (dotted line). The upward curvatures, i.e., convex profiles of $\ln M^F$ and $\ln P$ characterize a faster-than-exponential acceleration, associated with the positive feedbacks of price on money and of money on price. See text.

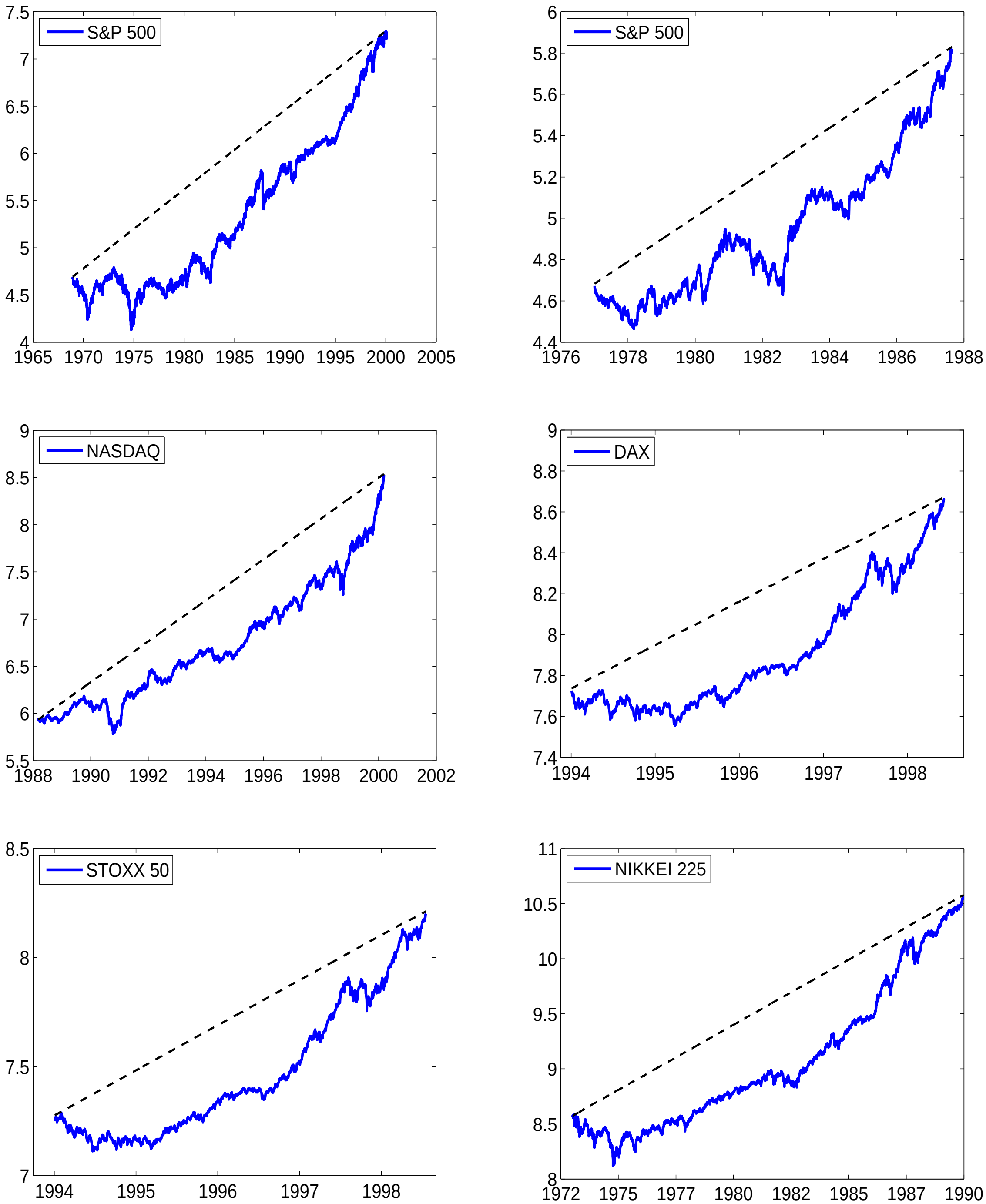


Figure 2: Empirical trajectories for the logarithms of the SP500 index from January 1965 to February 2000 (top left panel), the SP500 index from January 1977 to September 1987 (top right panel), the NASDAQ index from March 1988 to March 2000 (middle left panel), the DAX index from January 1994 to June 1998 (middle right panel), the STOXX 50 index from December 1986 to August 1991 (bottom left panel) and the NIKKEI 225 index from January 1973 to December 1989 (bottom right panel). In such a representation, a constant growth rate corresponding to a standard exponential growth is qualified by a straight line (dotted line). The upward curvatures characterize a faster-than-exponential acceleration, associated with the positive feedbacks of price on money and of money on price. See text.

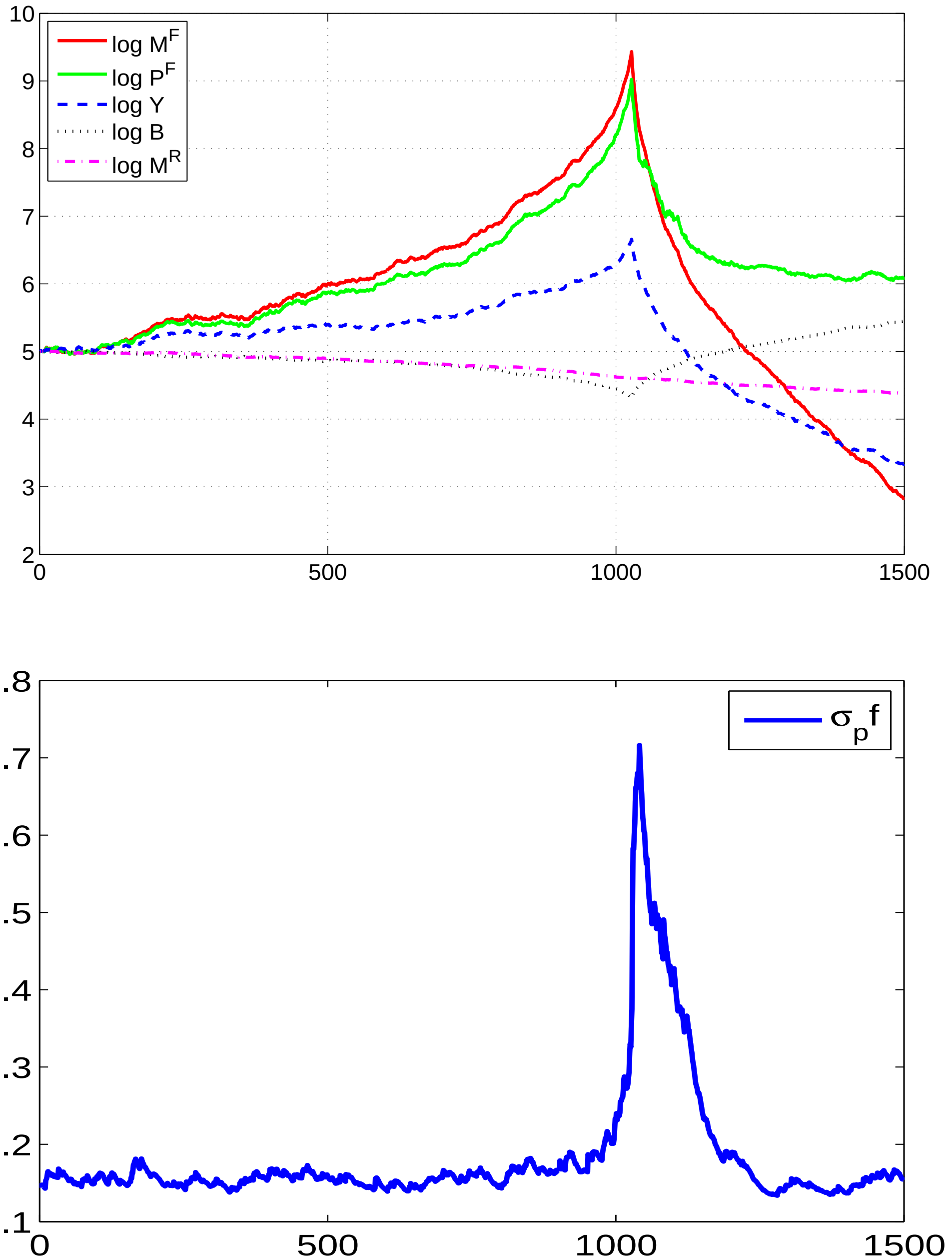


Figure 3: Top panel: Simulated trajectories with **deterministic regime switching** for money channeled to the financial sector M^F, financial prices P, real GDP Y, bankruptcy rate B and money channeled to the real sector M^R (variables are in normalized values). Bottom panel: Simulated path of the volatility of financial price $\sigma_{t,P}$.

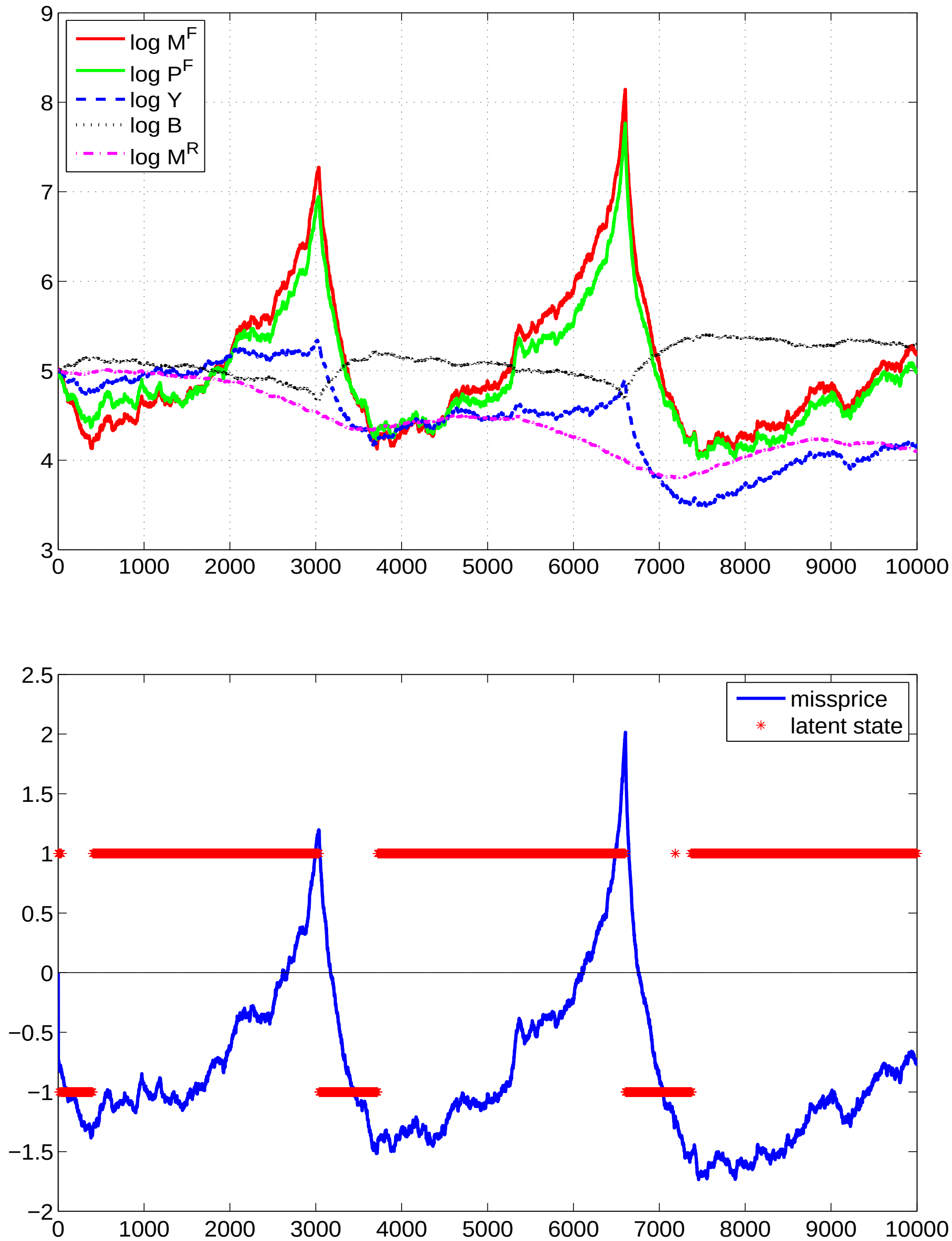
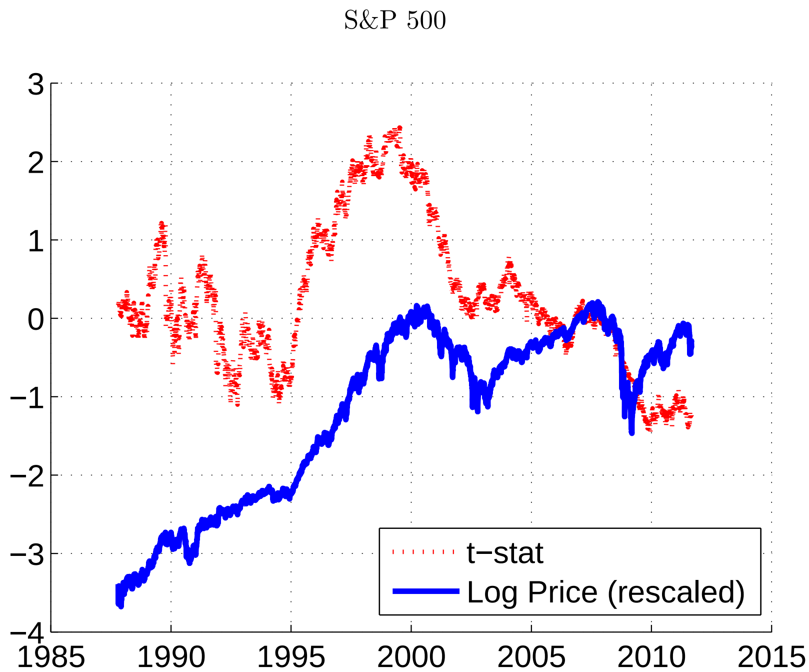


Figure 4: Top panel: Simulated trajectories with **stochastic regime switching** for money channeled to the financial sector M^F, financial prices P, real GDP Y, bankruptcy rate B and money channeled to the real sector M^R (variables are in normalized values). Bottom panel: Dynamics of the latent state variable s_t ($+1$ for $s_t = H$ when $\xi_t \geq 0$ and -1 for $s_t = L$ when $\xi_t < 0$, see Eq.(6)) and its triggering variable x_t represented by the financial asset misspricing.



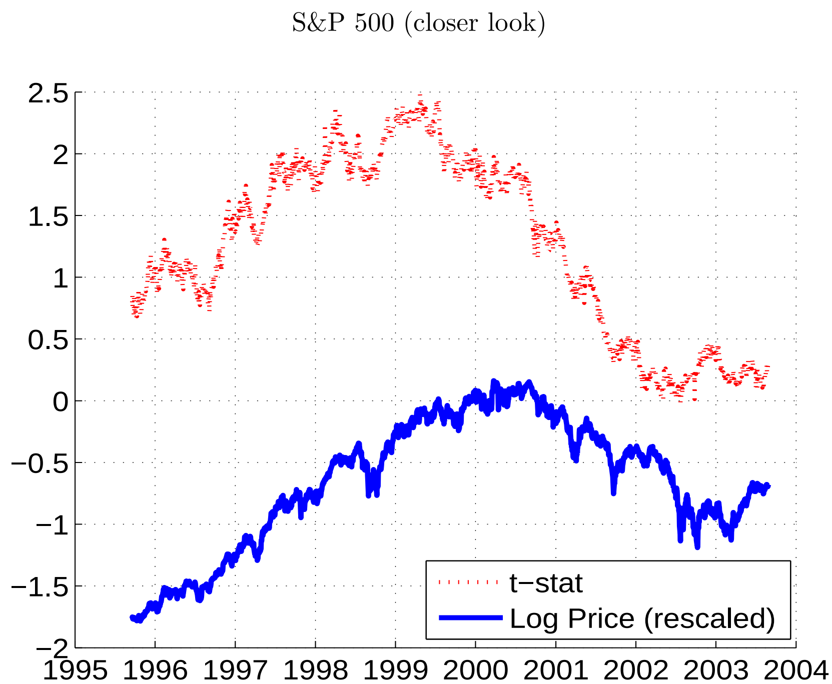
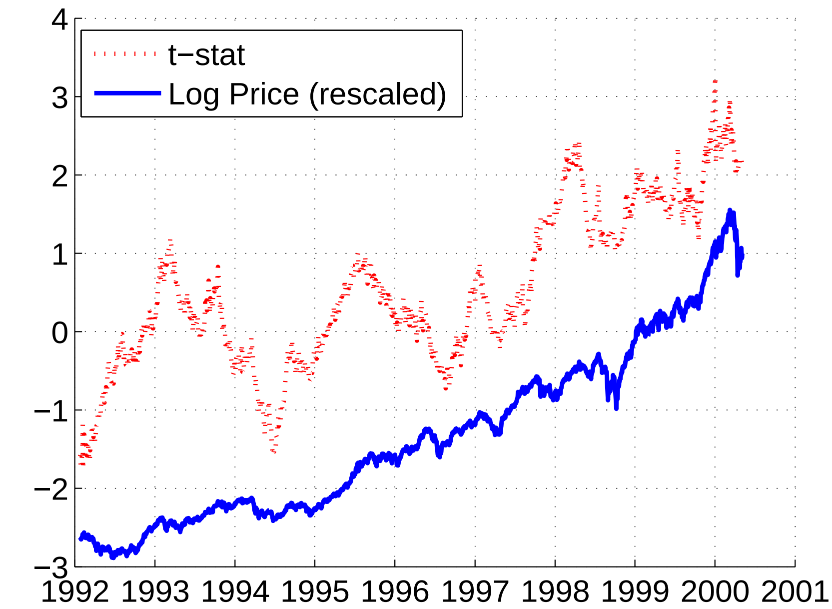


Figure 5: Top panels: Log price of S&P 500 October 1987 to September 2011 (8721 daily observations) and t-statistics of the FTS parameter γ computed on a moving window of 8000 days. Bottom panels: Close up from September 1995 to August 2003.

NASDAQ Composite



NASDAQ Composite (closer look)

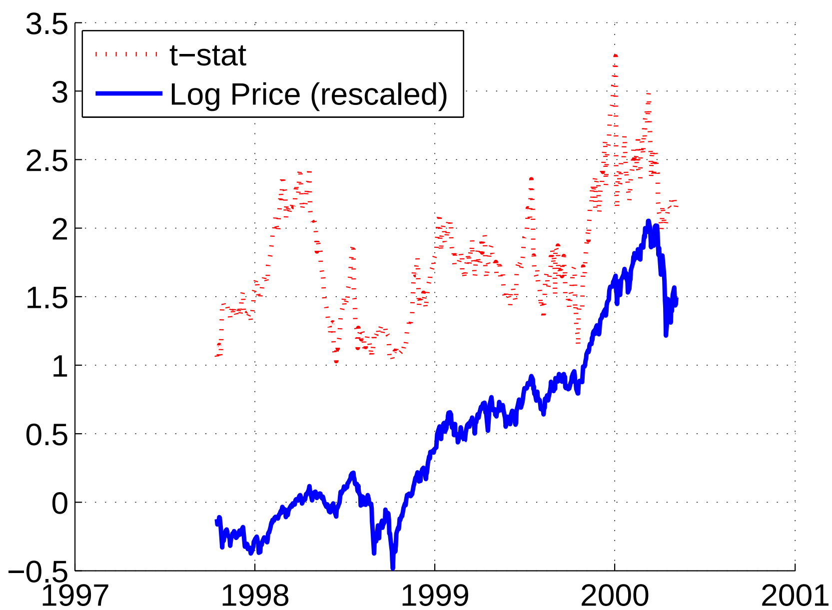
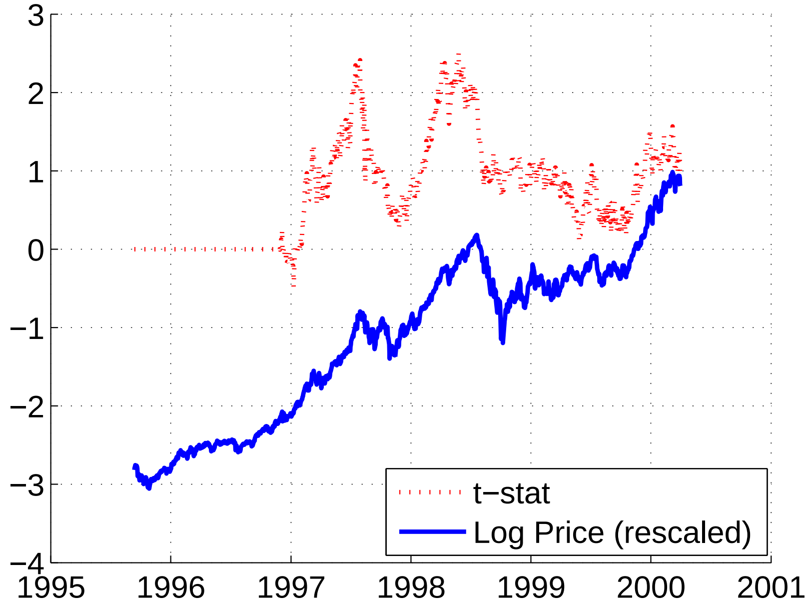


Figure 6: Top panels: Log price of NASDAQ Composite from January 1992 to May 2000 (3024 daily observations) and t-statistics of the FTS parameter γ computed on a moving window of 3000 days. Bottom panels: Close up from October 1997 to May 2000.

DAX



DAX (closer look)

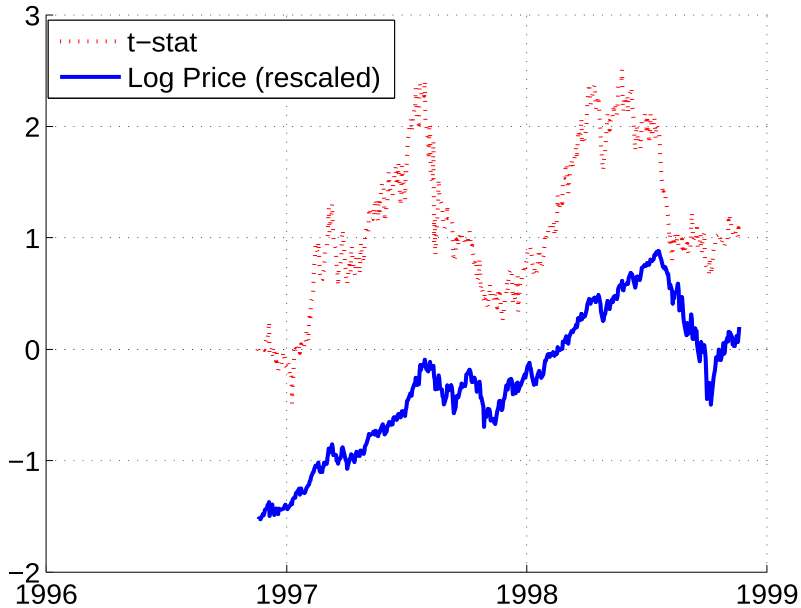
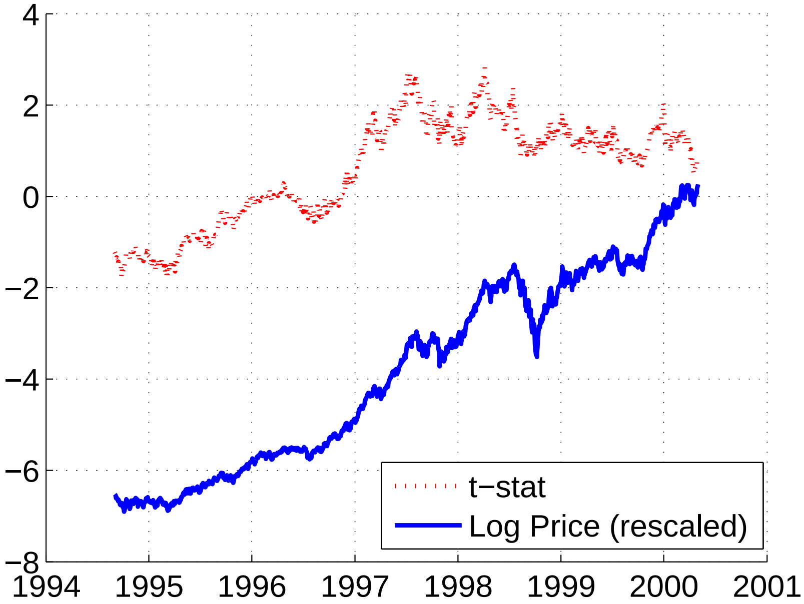


Figure 7: Top panels: Log price of DAX index from September 1995 to March 2000 (1662 daily observations) and t-statistics of the FTS parameter γ computed on a moving window of 1500 days (the zero values in the first part of the series correspond to the build up period of the moving window). Bottom panels: Close up from November 1996 to November 1998.

STOXX 50



STOXX 50 (closer look)

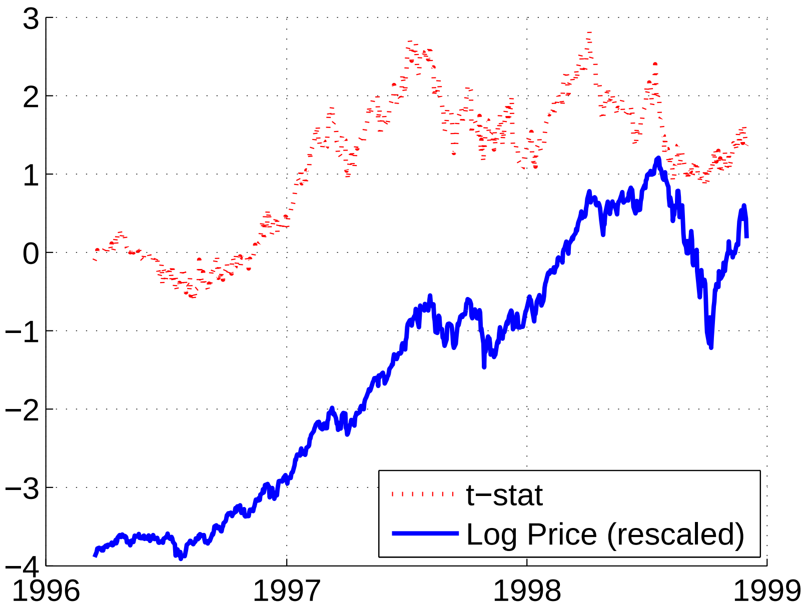
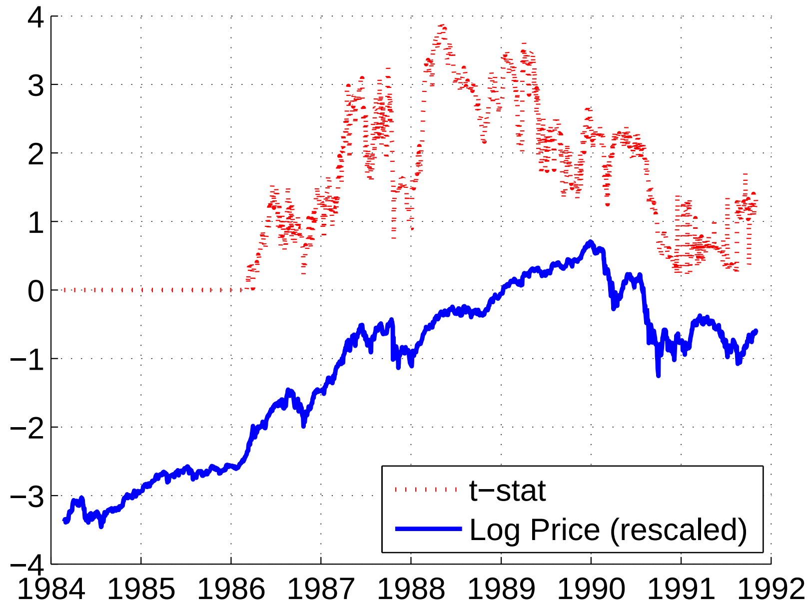


Figure 8: Top panels: Log price of STOXX index from September 1994 to May 2000 (2069 daily observations) and t-statistics of the FTS parameter γ computed on a moving window of 1500 days. Bottom panels: Close up from March 1996 to December 1998.

NIKKEI 225



STOXX 50 (closer look)

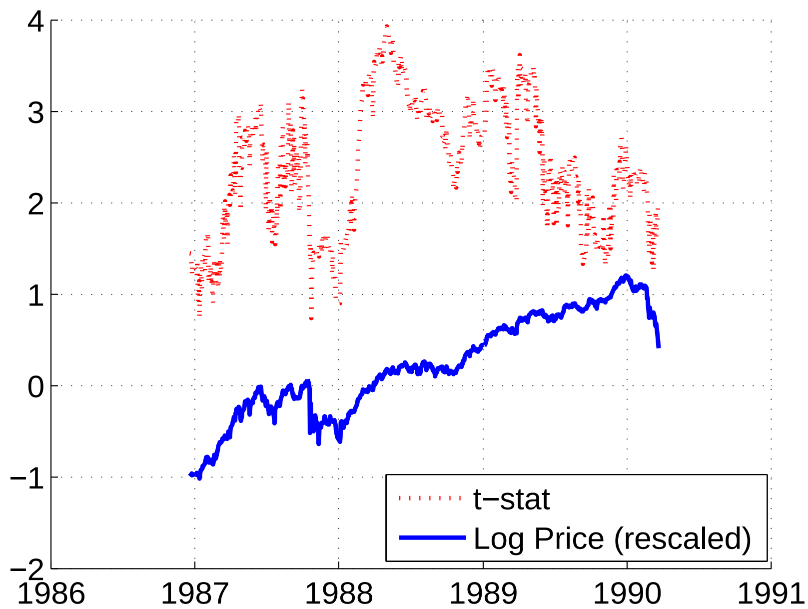


Figure 9: Top panels: Log price of NIKKEI 225 index from February 1984 to April 1992 (2960 daily observations) and t-statistics of the FTS parameter γ computed on a moving window of 4000 days (the extreme return on 19 October 1987 has been excluded form the sample). Bottom panels: Close up from December 1986 to March 1990.

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